Energy Efficiency in General Equilibrium with Input-Output Linkages

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Abstract

Industrial activity periodically experiences breakthrough innovations in energy efficiency, but the estimated impacts of these innovations on aggregate energy use are highly varied. We develop a general equilibrium model to show this variation is in part determined by the structure of the economy's input-output network. Our results show industrial energy efficiency improvements affect aggregate energy use through adjustments in factor and commodity markets, and a process of structural transformation that alters the way energy is used and produced in the economy. We link the aggregate impact of these processes with new energy centrality concepts that measure the extent to which an industry implicitly produces or consumes energy resources. In a calibrated simulation, we find variation in these energy centrality concepts explains between 38 and 92 percent of variation in the aggregate impacts of energy efficiency, which suggests input-output structure is a critical determinant of the aggregate effects of energy efficiency.

Keywords: energy efficiency, production network, input-output, rebound effect

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1 Introduction

Many of the proposed benefits from higher energy productivity are predicated on the notion that investments in energy efficiency reduce energy use. At the aggregate level, whether or not this holds true is an open question with a wide range of candidate estimates. What explains the wide variation in estimates, and how do we reconcile the differences? In the *The Efficiency Dilemma*, David Owens offers a potential solution:

> Teasing out the precise contribution of a particular efficiency improvement isn't just difficult; however, it may be impossible, because the endlessly ramifying *network of interconnections* is too complex to yield to empirical mathematics based analysis. (emphasis added)

Captured by the limitations of the time, theories of energy efficiency to-date bypass the complexity of the question by assuming away the intricate interconnections that support the economy's production system and are therefore limited in their ability to assess the contribution of an energy efficiency improvement to aggregate energy use. In this paper, we use recent advancements in macroeconomic theory to develop a general equilibrium model that incorporates input-output networks to analyze how the "endless ramifying network of interconnections" dictates the aggregate impacts of energy efficiency improvements.

To develop our argument, we begin by introducing a stylized general equilibrium model of the economy that embeds industrial, energy efficiency improvements within a disaggregated microeconomic production structure. The model combines elements from the energy efficiency literature (Chan and Gillingham 2015; Lemoine 2020) with workhorse models from the burgeoning literature on the macroeconomic implications of production networks (Acemoglu et al. 2012; Baqaee 2018). In the presence of input-output linkages, we find industrial energy efficiency improvements, modeled as an exogenous productivity shock to a sector's energy conversion technology, initiate a cascade of input reallocation across sectors as follows: The energy efficiency improvement lowers the marginal cost of production by reducing the energy service price for the sector experiencing the productivity shock. Under marginal cost pricing, a lower marginal cost is passed on as lower input prices to downstream industries in the network. Consequently, as the price shock propagates through downstream input-output linkages, the energy efficiency improvement spurs an expansionary process in these industries. If input substitution is sufficiently flexible, then producers can take advantage of lower priced intermediate inputs by substituting labor for these cheaper intermediates. As downstream industries expand their production, they also require more intermediate inputs from upstream industries. As downstream demand increases, upstream producers will absorb excess labor in the market and expand their own production to meet the intermediate requirements of downstream industries. We capture the richness of these interactions in our model.

To solve our model, we identify a mapping between an economy defined over the space of *energy service* inputs and an economy defined over the space of *physical energy* inputs. This mapping links the energy services used for the production of intermediates with the physical energy resources embedded in those services. We achieve this by expressing the service-based input-output network as a modified goods-based input-output network. Using this equivalence, we are able to apply comparative statics directly to our equilibrium solutions using standard input-output techniques and recover the general equilibrium responses to changes in the network structure.

To guide intuition, we introduce two new network centrality concepts: the *upstream-energy centrality* and *downstream-energy centrality*. These centrality concepts summarize an industry's implicit role as a producer or consumer of energy resources. Even if an industry does not directly supply inputs to the energy industry or consume energy intermediates, the industry may still be indirectly responsible for supplying inputs to or consuming from the energy sector in the presence of input-output linkages. Our energy-centrality concepts capture this possibility. Importantly, with input-output linkages, we find the energy efficiency shocks leads to a structural adjustment in the topology of the input-output network, and the extent to which the implicit upstream and downstream position of a sector adjusts following the energy efficiency shock depends on these energy centrality concepts. While equilibrium centrality concepts capture the capacity of a network to *transmit* a shock, the energy-centrality concepts allows us to characterize the capacity of a network to endogenously *adjust* to an energy efficiency shock. We introduce a novel analytical proposition that illustrates how idiosyncratic energy efficiency improvements alter the topological details of the economy's input-output linkages and show how the energy efficiency shock changes the implicit position of a sector as a producer or consumer of embodied energy.

We use our model to explain the mechanisms behind the general equilibrium "rebound effect." The rebound effect has interested economists since Jevons (1865) noted that, in a general equilibrium setting, adjustments in commodity and factor markets create behavioral responses that could entirely offset any potential energy savings from energy efficiency investments. Our main results show general equilibrium channels, namely, a price, scale, and composition effect, determine aggregate energy savings following an energy efficiency improvement. The price and scale effect account for the change in aggregate energy consumption spurred by economy-wide adjustments in factor and commodity prices. Yet, adjustments in market prices are only part of the general equilibrium rebound story. When sectors are linked within the economy's input-output network, we identify a composition channel that operates independently of price movements in commodity and factor markets.[1] This channel emerges because energy efficiency induces a structural shift in the industrial makeup of the economy, changing the way energy is used and produced in the economy. This result suggests energy efficiency improvements change the embodied energy in goods and services produced within the economy, resulting in a multiplier effect on partial equilibrium energy savings.

Most estimates of the general equilibrium rebound effect are derived from numerical methods, and due to heterogeneity in modeling assumptions, areas of study, and industry characteristics, general equilibrium rebound estimates exhibit substantial variation, generally ranging from 50 to 100% (Stern 2020).[2] We show how variation in the network of

[1] We use the term "composition" to capture the notion that the energy efficiency improvement changes the structural composition of the economy's input-output network. The change in the structural makeup of the economy in this sense is purely technological, where the more productive sector requires fewer energy inputs per dollar of output. In a networked-setting, this technological shift leads to a new network topology that is independent of price movements.

[2] Smaller or larger values can be found in the literature, and estimates overall range from negative rebound to backfire (Saunders 2013; Turner 2013; Gillingham et al. 2016). For instance, using a CGE model of the UK, Turner (2009) find rebound varies between -13% to 322%, Adetutu et al. (2016) find partial equilibrium rebound may go as low as -36% in the long run from estimates using panel data techniques, and Yu et al. (2015) find rebound ranges from -0.4% to 170% in a CGE model for the state of Georgia.

input-output linkages is sufficient to describe variation in estimated rebound across computational general equilibrium (CGE) applications. Even if an analyst adopts a common model framework, with identical elasticities of substitution, and applies this framework to two similar economies, differences in the network of input-output linkages across the economies will generate variation in estimated rebound. Moreover, variation in estimates can also emerge from the same shock applied to different sectors within the same economy. We show this final result is purely based on supply chain relationships embedded in the economy's input-output network and does not depend on the size or energy intensiveness of the sector experiencing the efficiency gain.

We complement our theoretical contributions by calibrating our model to input-output data collected for each US state. By incorporating input-output linkages into the model's structure, we find that we can simultaneously untangle the mechanics underlying the general equilibrium rebound effect and offer new explanations for the variation in available numerical estimates. Using the calibrated model, we simulate general equilibrium rebound effects by iteratively applying energy efficiency shocks to each sector in every state. The results of this exercise show general equilibrium rebound effects tend to be higher than the partial equilibrium prediction but also exhibit substantial variation.[3] We link variation in our estimates to assumptions regarding the elasticity of substitution and, more importantly, to variation in the topology of each state's input-output network. Our results suggest variation in the topology of the economy's input-output network explains between 38 to 92 percent of variation in our simulated rebound effects.

To introduce input-output networks, we build from a recent wave of empirical and theoretical research on the propagation of microeconomic shocks across input-output networks. Since the early contributions by Leontief (1936) and Hirschman (1958), many have argued the economy's input-output network serves as a mechanism for the transmission and amplification of idiosyncratic, microeconomic shocks, e.g. Acemoglu et al. (2012),

[3]Lemoine (2020) numerical exercise shows the dominance of the partial equilibrium effect in general equilibrium rebound, which scales linearly with an industry's elasticity of substitution. We conduct a similar exercise and find the *average* general equilibrium effect is dominated by and scales linearly with the partial equilibrium effect. However, for the same elasticity of substitution, we find there is still substantial variation within these estimates and show this variation can be explained by differences in the topology of the input-output network.

Acemoglu et al. (2016), Barrot and Sauvagnat (2016), Atalay (2017), Baqaee (2018), Baqaee and Farhi (2019), Boehm et al. (2019), Liu (2019), and King et al. (2019). In this paper, we extend this literature by studying the effects of energy-augmenting productivity shocks on aggregate energy use. Moreover, unlike these studies, we analyze the model from the perspective of a factor-augmenting productivity shock that only affects a sector's energy input-output coefficient, as in Hogan and Jorgenson (1991).[4]

Our model is closely related to Baqaee (2018), featuring non-unitary elasticities of substitution, multiple sectors, and input-output linkages. Since the early contribution of Saunders (1992), economists are aware that the magnitude of the general equilibrium rebound effect is sensitive to the choice of elasticity of substitution between energy and non-energy inputs to production. Early theoretical contributions made this point clear through the lens of single-sector, neo-classical models (Saunders 2008; Wei 2010), where a substitution effect dictated by microeconomic elasticities of substitution determines the magnitude of the general equilibrium rebound effect.[5] These models predict that, when inputs are gross substitutes, energy efficiency improvements cause aggregate energy use to increase (backfire), and vice versa. By incorporating non-unitary elasticities of substitution in our model, we find a similar result for the partial equilibrium effect, but importantly, we find the magnitude of the partial equilibrium effect is amplified by the presence of input-output linkages. Moreover, we link the magnitude of amplification with the topology of the economy's input-output network.

Recent work by Hart (2018), Böhringer and Rivers (2018), an Lemoine (2020) extend the analytical literature to the case of multiple consumption goods. In terms of underlying mechanisms, these papers uncover additional channels through which economy-

[4]As noted in Baqaee and Farhi (2019), factor-augmenting productivity shocks can be re-cast as Hick's neutral productivity shocks by re-structuring input relationships into "fictitious" sectors. Although the theoretical implications are the same between the two modeling environments, we elect to solve the model using the factor-augmenting theoretical formulation. We note our modeling choice is consistent with how industrial, energy efficiency shocks are modeled in the rebound literature.

[5]Our model stands in contrast with Lemoine (2020), where heterogeneous elasticities of substitution, in both production and consumption, dictate economy-wide rebound. We elect to the follow the modeling procedure in Baqaee (2018) and abstract away from heterogeneity in substitution elasticities for two reasons. First, this abstraction allows us to solve a complicated model framework in closed-form. The second reason is that the elasticity of substitution's, even if heterogeneous, contribution to the magnitude of the general equilibrium rebound effect is well known. Thus, we instead focus our efforts on studying a new source of variation, i.e. the topology of the input-output network.

wide rebound effects arise. Using an expanding varieties model, Hart (2018) find evidence that the substitution and scale effect have increased demand for energy-intensive goods. Böhringer and Rivers (2018) develop a model with two consumption goods and illustrate that general equilibrium rebound can arise through a substitution effect, scale effect, composition effect, and labor supply effect. Lemoine (2020) finds the same set of channels in addition to an energy supply effect in a model with multiple sectors and heterogeneous, non-unitary elasticities of substitution. Importantly, the energy supply effect emerges from an input linkage, where the energy sector uses its own output as an input to production.[6] Unlike these papers, in our model, a firm's production technology not only uses labor and energy inputs, but also non-energy intermediate inputs. This key difference in modeling assumptions implies previous models are unable to account for how energy efficiency shocks propagate throughout the economy and change the underlying input-output structure of the production system. We show how this has important implications for rebound and the non-energy sectoral incidence of productivity shocks, since producer's may indirectly supply inputs to or consume output from the energy sector.

The organization of the paper is as follows. Section 2 introduces the economic environment and highlights the key features of our model. In section 3, we present the equilibrium solutions to the model and link equilibrium prices and quantities with static network centrality concepts. Section 4 presents the main results of the paper. By applying comparative statics to the model's equilibrium, we show how equilibrium aggregate energy savings from energy efficiency improvements are determined by the topology of the input-output network. Section 5 takes the model to data to evaluate the model's predictions. Section 6 summarizes the main insights of the paper and offers potential avenues for future research.

[6] The energy supply effect in (Lemoine 2020) is a special case of our composition effect. If we remove the input-output linkages between the energy sector and non-energy sectors, our model will reproduce the same general equilibrium effect.

2 The Model

We consider a static model with two types of agents, a representative consumer and industrial producers. The representative consumer maximizes utility over an exogenous, discrete set of consumption goods and services. This consumption set is divided into an energy commodity c_e and $N-1$ non-energy commodities c_i. The representative consumer inelastically supplies a fixed labor endowment of $\bar{L}$ and collects income $C = w\bar{L}$, where w is the wage rate.

Each producer in the model corresponds to a sector. Each sector's production technology uses labor supplied by the representative consumer and intermediate inputs sourced from other sectors in the economy. Producers choose input bundles to minimize the total cost of production. Production in the N sectors is allocated to goods and services for both intermediate use in other sectors and final-use consumption by the representative household. We assume markets for goods and services are perfectly competitive.

2.1 Preferences

The representative consumer's preferences are modeled using a constant-elasticity of substitution (CES) utility function U defined over $i \in \{e, 2, \ldots, N\}$ industrial products, where the energy commodity e is in the first index. The representative consumer maximizes utility choosing over consumption levels c_i according to the following program

$$\begin{aligned} \max_{\{c_e, c_2, \ldots, c_N\}} U(c_e, c_2, \ldots, c_N) &= \left[\alpha_e^{\frac{1}{\sigma}} c_e^{\frac{\sigma-1}{\sigma}} + \sum_{i \neq e}^{N-1} \alpha_i^{\frac{1}{\sigma}} c_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ \text{s.t.} \quad C &= p_e c_e + \sum_{i \neq e}^{N-1} p_i c_i \end{aligned} \tag{1}$$

where $\sigma > 0$ is the household's elasticity of substitution, the parameters $\alpha_i \geq 0$,$\forall i \in \{e, 2, \ldots, N\}$ capture the representative consumer's tastes for goods and services produced in the economy, p_i is the price of sector i's product, and C is the income of the consumer. Consumers choose a consumption plan $\mathbf{c}$ to maximize utility U according to the constrained maximization problem in (1). The household takes the wage rate w and labor endowment $\bar{L}$ as given. Household demand for good i is expressed as

$$c_i = \alpha_i \left(\frac{p_i}{P_h} \right)^{-\sigma} \frac{C}{P_h} \tag{2}$$

and it is determined by consumer preferences, product prices, and household income. As it is standard, equation (2) implies household demand for good i will increase when product prices decline or household incomes rise.

We choose the consumer price index, P_h, which measures the cost of purchasing one unit of utility, as the numeraire of the economy so that all prices and income are expressed in real terms. The consumer price index P_h is given by

$$P_h = \left(\alpha_e p_e^{1-\sigma} + \sum_{i \neq e}^{N-1} \alpha_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} = 1 \tag{3}$$

2.2 Production

Producers use a constant returns-to-scale CES production technology to produce goods or services. Each sector i (the purchasing sector) combines intermediate inputs x_{ji} from other industries j (the supplying sectors) with labor L_i provided by the representative household. Sector i's production technology is characterized as[7]

$$y_i = \left[\gamma_i^{\frac{1}{\sigma}} L_i^{\frac{\sigma-1}{\sigma}} + \omega_{ei}^{\frac{1}{\sigma}} \left(\phi_{ei} x_{ei} \right)^{\frac{\sigma-1}{\sigma}} + \sum_{j \neq e}^{N-1} \omega_{ji}^{\frac{1}{\sigma}} x_{ji}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{4}$$

where γ_i is a distribution parameter measuring the labor intensiveness of sector i. We assume each sector combines a *physical energy input* (x_{ei}) with an *energy conversion technology* (ϕ_{ei}) to produce an *energy-service* ($x_{si} = \phi_{ei} x_{ei}$). The parameter ϕ_{ei} directly measures the

[7] We model production without the use of capital inputs to facilitate the interpretation of our main results. In the Appendix D, we present an alternative specification for production where L_i in (4) is replaced by a value added composite $V(K_i, L_i)$, where K_i is the capital input. When V is Cobb-Douglas, we show our main equilibrium solutions are proportional up to a constant scalar amount to the equilibrium with capital inputs. Because the capital share is constant within the Cobb-Douglas framework, the proportional results imply the mechanics of the model with and without capital are the same. Importantly, however, we take care to note the inclusion of capital inputs introduces a new source of variation for general equilibrium rebound. In this appendix, we illustrate how the magnitude of rebound inherently depends on the capital share of production and the size of the per capita capital stock.

productivity of a sector's energy conversion technology and variations in this parameter are the source of energy efficiency improvements in the model. Throughout the remainder of the paper, we will refer to the sector experiencing the energy efficiency improvement as the "source sector" and, where it does not cause confusion, we label the source sector as i. Let $\boldsymbol{\phi}$ be an $N \times N$ matrix of these productivity parameters, where the first row corresponds to the energy productivity parameters of each sector and the remainder of entries in the matrix are equal to 1.[8]

The parameters ω_{ei} and ω_{ji} for $i \in \{e, 2, \ldots, N\}$ are the technical input-output coefficients of sector i and, collectively, these coefficients define the structure of the intermediate production network of the economy (Acemoglu et al. 2012; Baqaee 2018). The input-output network of the economy is represented by the $N \times N$ matrix $\boldsymbol{\Omega}$ of these input-output coefficients.[9]

We assume that firms in each industry minimize the costs of production subject to the available production technology given in equation (4). Given the exogenous labor share parameter γ_i and the input-output coefficients ω_{ji}, firms in sector i choose labor L_i and intermediate inputs $\{x_{ei}, x_{2i}, \ldots, x_{Ni}\}$ to solve the following cost minimization problem

$$\min_{\{L_i, x_{ei}, x_{2i}, \ldots, x_{Ni}\}} wL_i + p_e x_{ei} + \sum_{j \neq e}^{N-1} p_j x_{ji} \tag{5}$$

subject to (4), exogenous energy conversion productivity parameters ϕ_{ei}, the input-output network $\boldsymbol{\Omega}$, the economy's wage rate, w and the market price for sector i's output, p_i.

After solving the producer's minimization problem, sector i's demand for energy and non-energy intermediates are

$$x_{ei} = \left(\frac{\omega_{ei}}{\phi_{ei}}\right) \left(\frac{p_{si}}{\mu_i}\right)^{-\sigma} y_i \tag{6a}$$

$$x_{ji} = \omega_{ji} \left(\frac{p_j}{\mu_i}\right)^{-\sigma} y_i \tag{6b}$$

[8] This structure is sufficient for the objectives of this paper, the structure of $\boldsymbol{\phi}$, however, can be extended to capture many other technological improvements in the economy that impact the input-output relationships between industries.

[9] This is also sometimes referred to as the direct requirements matrix.

Sector i's demand for energy x_{ei} depends on several quantities of interest. First, intermediate demand for energy will be determined by sector i's direct energy service requirements, $\left(\frac{\omega_{ei}}{\phi_{ei}}\right)$, which is proportional to energy intensiveness. Second, demand for energy inputs will depend on the price of energy services $p_{si} = {}^{p_e}/_{\phi_{ei}}$ and the marginal cost of production μ_i. Third, intermediate demand for energy will vary with production levels y_i.

3 Equilibrium

The economic environment introduced in Section 2 allows us to relate equilibrium commodity prices $\mathbf{P}$, the wage rate w, and production levels $\mathbf{y}$ to two well-known, equilibrium network centrality concepts. In this section, we illustrate how equilibrium prices in the economy are determined by a sector's *consumer centrality* and output levels are determined by a combination of *consumer* and *supplier centrality*.

3.1 Definition

We say the economy is in equilibrium when households maximize utility subject to their income constraint, producers minimize costs within a perfectly competitive environment, and commodity and factor prices clear the markets.

Definition 1. (General Equilibrium) *A general equilibrium in the economy $\mathcal{E} = (\mathbf{P}, w, \mathbf{X}, \mathbf{y}, \mathbf{c}, \boldsymbol{L})$ is characterized by an $N \times 1$ vector of output prices $\mathbf{P}$, an economy-wide wage rate w, an $N \times N$ matrix of intermediate demand $\mathbf{X}$, an $N \times 1$ vector of total output $\mathbf{y}$, an $N \times 1$ final-use consumption plan $\mathbf{c}$, and an $N \times 1$ vector of labor demand $\mathbf{L}$, such that the following conditions are met:*

1. *Given the $N \times 1$ vector of taste parameters $\boldsymbol{\alpha}$, the consumption plan $\mathbf{c}$ maximizes utility U subject to the consumer's budget constraint $C = w\bar{L}$*

2. *Given exogenously determined productivity parameters $\boldsymbol{\phi}$, the input-output network $\boldsymbol{\Omega}$, and labor intensities $\boldsymbol{\gamma}$, the production plan given by the vector of total output $\mathbf{y}$, the matrix*

of intermediate demand $\mathbf{X}$*, and the vector of labor demand* $\mathbf{L}$ *minimize the total costs of production for each sector and are technologically feasible.*

3. *Markets for each good and the labor market clear so that* $\mathbf{y} = \mathbf{X}\boldsymbol{\iota} + \mathbf{c}$ *and* $\bar{L} = \mathbf{L}\boldsymbol{\iota}$*, where* $\boldsymbol{\iota}$ *is an* $N \times 1$ *vector of ones.*

4. *Perfect competition implies firms price at marginal cost and earn zero economic profits.*

Next, we introduce the concept of a *goods-based input-output network*. The goods-based input-output network allows for technological improvements, in our case energy efficiency improvements, to be expressed as changes in the direct requirements of physical units of the energy input rather than units of the energy service. This concept is useful since most, if not all, input-output tables in practice are measured in terms physical quantities of inputs, rather than service inputs. Furthermore, we can vary these input-output parameters since they are proportional to the productivity parameter ϕ_{ei} and the exogenous input-output parameter ω_{ei}. Our procedure is similar to the method for adjusting price indexes for changes in the underlying quality of goods consumed, e.g. see Feenstra (1995). The following definition establishes the relationship between the service-based input-output network $\mathbf{\Omega}$ and the goods-based input-output network.

Definition 2. (Goods-Based Input-Output Network) *The goods-based input-output network relates the production technology given in (4) to a production technology defined in the space of physical energy inputs rather than energy service inputs. The goods-based input-output coefficient for energy inputs is given by*

$$\omega^*_{ei} = \phi^{\sigma-1}_{ei}\omega_{ei} \tag{7}$$

and the goods-based input-output network is given by

$$\mathbf{\Omega}^* = \boldsymbol{\phi}^{\sigma-1} \odot \mathbf{\Omega} \tag{8}$$

where the exponent represents element-wise exponentiation and the character $\odot$ *is the Hadamard product.*

Definition 2 states that the production technology given in (4) is consistent with an input-output network defined over the space of energy service requirements. While the *service-based* input-output network captures the embedded energy services required to produce output, the *goods-based* input-output network reflects the actual amount of physical goods required for production. In our setting, the goods-based input-output network reflects the embodied energy requirements necessary to produce output in the economy.

For the remainder of the paper, we will use $\mathbf{\Omega}^*$ to denote the *goods-based* input-output matrix and make the following assumption regarding the entries in the matrix

Assumption 1. *The productivity of a sector's energy conversion technology ϕ_{ei} is sufficiently small so that all (i, j) entries of $\mathbf{\Omega}^*$ satisfy $|\omega^*_{ij}| < 1$.*

Assumption 1 is innocuous in the context of energy efficiency. Within this context, we can scale the units of the energy conversion technology to ensure ϕ_{ei} is sufficiently small for all sectors.[10] Ultimately, the assumption is necessary for the power series expansion of the Leontief inverse to converge to a finite quantity. With Assumption 1 satisfied, we define the economy's multiplier matrix as follows:

Definition 3. (Multiplier Matrix) *The multiplier matrix $\mathbf{M}$ is an $N \times N$ matrix given by*

$$\mathbf{M} = [\mathbf{I} - \mathbf{\Omega}^*]^{-1} \tag{9}$$

and, given Assumption 1 holds, $\mathbf{I} - \mathbf{\Omega}^*$ is non-singular.

The multiplier matrix $\mathbf{M}$ has the same interpretation as the Leontief inverse matrix in input-output analysis (Leontief 1936; Miller and Blair 2009) and accounts for all direct and indirect interactions between sectors in the economy.

3.2 Equilibrium Prices, Quantities, and Wages

In this section, we connect equilibrium prices and quantities to underlying characteristics of the network of input-output linkages. The following definition allows us to interpret

[10] For example, if ϕ_{ei} measures vehicle energy efficiency and is expressed in miles per gallon, then there is some constant c such that ϕ_{ei}/c is expressed in miles per c gallon that satisfies Assumption 1. For instance, setting $c = \sup\{\phi_{ee}, \phi_{e1}, ..., \phi_{eN}\}$ would satisfy Assumption 1.

our equilibrium results using the topological details of the economy's input-output network (Baqaee 2018):

Definition 4. (Equilibrium Network Centrality Concepts) *The following equilibrium centrality concepts relate a sector's systemic importance as either a consumer of factor inputs or supplier of final goods to its Bonacich centrality.*

1. *The consumer centrality of a sector measures its systemic importance as a direct or indirect purchaser of factor inputs in the economy. The vector of consumer centralities is defined as*

$$\boldsymbol{\Delta} = \mathbf{M}^{'}\boldsymbol{\gamma} \tag{10}$$

2. *The supplier centrality of a sector measures its systemic importance as a direct or indirect supplier of final goods in the economy.*[11] *The vector of supplier centralities is defined as*

$$\mathbf{Y} = \mathbf{M}\boldsymbol{\alpha} \tag{11}$$

Both $\boldsymbol{\Delta}$ and $\mathbf{Y}$ are $N \times 1$ vectors of Bonacich (1987) centralities. Larger centrality values imply a sector occupies a more "central" position in the economy's production network as a purchaser of factor inputs or supplier of final goods. Consumer centrality depends on a sector's own consumption of factor inputs, as well as factor input use of direct and indirect upstream suppliers. Formally, the consumer centrality of a sector is given by

$$\boldsymbol{\Delta}_j = \gamma_j + \sum_k \omega_{kj}\boldsymbol{\Delta}_k$$

In heterogeneous production networks, some sectors may be more susceptible to price shocks because of their more central role as a purchaser of inputs in the economy, and the above expression illustrates how this depends on topological details of the network. Similarly, supplier centrality reflects a sector's direct and indirect role as a supplier of

[11] Our choice of notation here is deliberate. As consumer centrality also captures the "downstreamness" of a particular sector, we elect to use "Delta," $\boldsymbol{\Delta}$, to reinforce this intuition to the reader. Similarly, The supplier centrality of a sector also reflects a sector's general "upstreamness" in the production network. To remind the reader of this intuition, we use "Upsilon," $\mathbf{Y}$, to capture this general notion.

final goods to the household and depends on the sector's own final use supply as well as the final use supply of all downstream producers. The supplier centrality of a sector is equivalently expressed as

$$\mathbf{Y}_j = \alpha_j + \sum_k \omega_{jk}\mathbf{Y}_k$$

Larger values suggest these sectors are influential in the supply of intermediate inputs in the economy, which are ultimately used to produce final goods for the representative household. As before, this measure of importance is grounded in the topology of the input-output network. In the next proposition, we show how the consumer and producer centrality measures relates to equilibrium prices and quantities:

Proposition 1. (Network Centralities and Equilibrium Solutions) *With the equilibrium network centrality concepts defined, we are able to solve for closed-form solutions for general equilibrium prices, output, and wages. The following expressions formally relate these concepts.*

1. *Equilibrium commodity prices are represented as*[12]

$$\mathbf{P} = (\boldsymbol{\Delta})^{\frac{1}{1-\sigma}}\, w \tag{12}$$

2. *The equilibrium sales vector is characterized as*

$$(\mathbf{P}^{\sigma} \odot \mathbf{y}) = \mathbf{Y}C \tag{13}$$

3. *The economy's equilibrium wage rate is given by*

$$w = \frac{C}{\bar{L}} = \left(\mathbf{Y}'\gamma\right)^{\frac{1}{\sigma-1}} = \left(\boldsymbol{\alpha}'\boldsymbol{\Delta}\right)^{\frac{1}{\sigma-1}} \tag{14}$$

The expression for prices in (12) illustrates the relationship between equilibrium prices and the network of input-output linkages. Specifically, the model predicts that consumer centrality plays an important role in determining prices in equilibrium, and this relationship depends on the elasticity of substitution in the economy. When production processes

[12]The exponent in this relation represents element-wise exponentiation.

approach the Leontief limit ($\sigma \to 0$), the expression predicts that prices will be higher in sectors with higher consumer centrality.

The intuition underlying this prediction follows from the theory of cost pass-through (Weyl and Fabinger 2013). In vertical supply chains, an upstream producer's prices are a downstream producer's costs, and price shocks in upstream markets will propagate through downstream input-output linkages only to the extent these price variations are passed on to downstream industries. The rate at which these price variations are passed through to downstream buyers is proportional to the price elasticity of demand for the upstream producer's products. If there is less scope for substitution, then sectors with a more central role as downstream purchaser of inputs will be more exposed to the price shock than less central sectors.

The expressions for equilibrium output in (13) contain three useful predictions for evaluating how energy efficiency shocks generate aggregate energy savings. First, the model predicts how higher output prices reduce sector output levels in equilibrium. The intuition for this result is straightforward and follows from the law of demand, where higher output prices reduce the quantity demanded for a sector's product. The second prediction of the expression for equilibrium output levels implies production levels in the economy are positively related with the income-level in the economy. Because there is no savings in the model, consumers exhaust their income on purchasing goods and services in the economy. When incomes increase, consumer demand for final goods and services will increase. The final prediction relates to a sector's upstream position in the economy's production network. If a sector is more essential for supplying final goods to the household, i.e. has a higher supplier centrality, the model predicts production will be higher in this sector.

The expression (14) characterizes equilibrium in the labor market. The expression stipulates the amount of labor embodied in the supply of goods and services must equal the amount of labor embodied in final goods and services consumed by the representative household.

4 The General Equilibrium Rebound Effect

In this section, we illustrate how the characteristics of the economy's network of input-output linkages determines the magnitude of the general equilibrium rebound effect. We start by applying an energy efficiency shock $d\phi_{ei}$ to sector i, which will hereafter be referred to as the *source sector*. After applying the efficiency shock, we establish how the shock propagates across the network of input-output linkages and impacts the implicit position of a sector as a producer or consumer of embodied energy inputs. We introduce two new network centrality concepts to encapsulate this intuition, i.e. *upstream-* and *downstream-energy centrality*.

Using these network concepts, we turn to how the energy efficiency shock elicits a cascade of reallocation across the input-output network. We show how this reallocation process manifests as the three main general equilibrium channels for rebound: a price, scale, and composition effect. We link the magnitude of energy savings from each channel with the topology of the input-output network. The final step establishes the main theoretical results that illustrate how the general equilibrium rebound effect varies with the characteristics of the input-output network.

To begin our analysis, we formalize how changes in the service-based production technology given in (4) relate to changes in the goods-based input-output network. In particular, we show that changes in the energy services used in production is equivalently expressed as changes in the goods-based energy input-output coefficient in the following proposition.

Proposition 2. (Mapping between Energy Services and the Goods-Based Network) *Variation in the consumption of energy services caused by an energy efficiency improvement directly maps into variation in the goods-based input-output network. Formally, this mapping is determined by*

$$(\sigma - 1) \frac{\partial x_{si}}{\partial \phi_{ei}} \frac{\phi_{ei}}{x_{si}} = \frac{\partial \omega_{ei}^{*}}{\partial \phi_{ei}} \frac{\phi_{ei}}{\omega_{ei}^{*}} \tag{15}$$

In the energy service space, the left-hand side of equation (15), when an industry becomes more energy efficient, the amount of energy services responds endogenously through changes in relative input prices and is mediated by the elasticity of substitution. In the equivalent goods-based space, the right-hand side of equation (15), the technical rate of substitution adjusts endogenously, changing the slope of the goods-based production technology's isoquant. The effects of the energy efficiency shock can then be equivalently expressed as either a change in the energy service intensity of a sector or a change in the topology of the goods-based input-output network.

The power behind Proposition 2 is that it allows us to view energy efficiency improvements through the lens of changes within the goods-based input-output network. When ω^*_{ei} adjusts, the entire goods-based input-output network will adjust in proportion to the change in energy intensiveness of the source sector $\frac{\partial \omega^*_{ei}}{\partial \phi_{ei}}$ resulting in a change in the way physical energy flows throughout the production system. Because sectors are connected within the economy's input-output network, the endogenous adjustment in the good-based input-output network cascades across the network and, therefore, alters the architecture of the economy, leading to more (or less) energy embodied per unit of output than before.

We make a deliberate distinction between input-output *architecture* and *topology*.[13] The input-output architecture adjusts because the energy efficiency improvement directly alters the direct input requirements in the goods-based input-output network. The change in input requirements results in either more or less energy used in the production of the source sector's product. In this sense, the architecture of the network is consistent with the notion of how energy resources are deployed in production of goods and services. The topology of the network, in contrast, governs the flow of energy resources throughout the economy. We summarize this insight in the following proposition.

Proposition 3. (Propagation and Network Architecture) *The energy efficiency improvement in the source sector propagates across the goods-based network of input-output linkages and*

[13]For more detailed discussion on the architecture versus topology distinction, see Oberfield (2018).

changes the network architecture according to

$$\frac{d\mathbf{M}}{d\phi_{ei}}\phi_{ei} = \mathbf{M}\frac{\partial\mathbf{\Omega}^*}{\partial\phi_{ei}}\mathbf{M} = \sum_{k=0}^{\infty}\frac{\partial\mathbf{\Omega}^{*k}}{\partial\phi_{ei}} \tag{16}$$

where the exponent k is a matrix power and captures the k^{th} order impact of the energy efficiency shock.

Proposition 3 provides the mathematical apparatus for interpreting how the topology of the input-output network drives variation in the economy-wide rebound effect. The first equivalence in the proposition sets the stage for how the structure of $\mathbf{M}$ fits into estimates of general equilibrium rebound, while the last equivalence shows how the shock will propagate along input-output linkages. These results allow us to isolate the impact of the input-output network from other details of the model, such as the elasticity of substitution and the source sector's energy intensiveness.

A broader interpretation of Proposition 3 is that an idiosyncratic energy efficiency shock induces structural change in the economy's underlying network of input-output linkages. From this structural transformation, the architecture of the economy endogenously adjusts in unison, where either more or less energy is required throughout the production system. Importantly, as the input-output network adjusts, following Proposition 3, the supplier and consumer centrality concepts are no longer suitable for summarizing a sector's contribution to market fluctuations. To address this, we introduce two network centrality concepts that account for when the network is perturbed by the efficiency shock.

Energy centrality concepts measure the extent to which a sector's relative network position after the network is perturbed by the energy efficiency shock. The following definition introduces these centralities

Definition 5. (Energy Centralities)

1. *The downstream-energy centrality measures the extent to which a sector implicitly uses energy resources in production. A sector's downstream-energy centrality is defined as*

$$\delta_{ij} = \omega^*_{ei}\mathbf{M}_{ij} \tag{17}$$

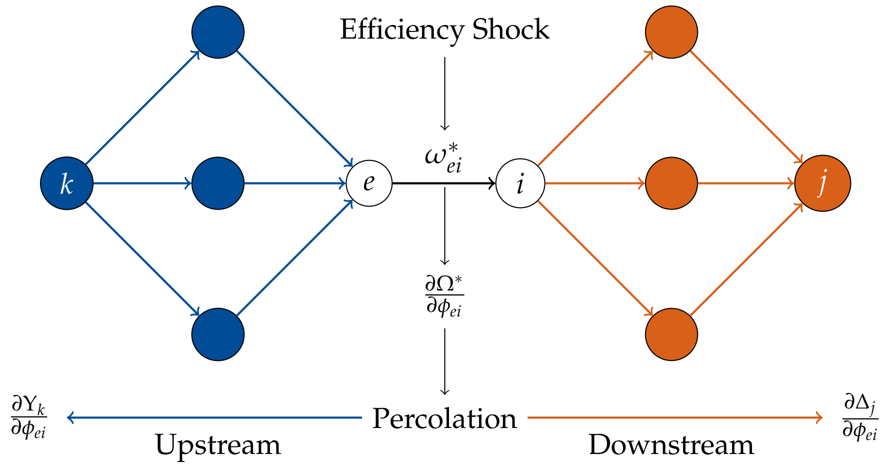


Figure 1: Energy Centralities. The energy efficiency shock $\partial\phi_{ei}$ adjusts the direct, energy requirements coefficient ω^*_{ei}, which changes the network state. This perturbation, captured by changes in the input-output matrix $\partial\mathbf{\Omega}^*/\partial\phi_{ei}$, initiates an upstream an downstream percolation process. This percolation process is affected by the implicit position of a sector as a producer or consumer of energy inputs. Energy centralities encapsulate both how and why k's and j's position adjusts after the energy efficiency improvement.

2. *The upstream-energy centrality measures the extent to which a sector is implicitly involved in the production of energy resources. A sector's upstream-energy centrality is defined as*

$$v_j = \mathbf{M}_{je} \tag{18}$$

We visually illustrate these concepts in Figure 1. The intuition for downstream-energy centrality follows from how consumer centrality $\mathbf{\Delta}_j$ changes after the energy efficiency shock. The percolation of the shock is shown in sectors downstream (to the right) of the source sector in Figure 1.[14] Given Proposition 3, we can write

$$\frac{\partial \mathbf{\Delta}_j}{\partial \phi_{ei}} \phi_{ei} = (\sigma - 1)\, \mathbf{\Delta}_e \delta_{ij}$$

The downstream-energy centrality captures the indirect ways j relies on energy resources

[14] We are deliberate in our use of the word "percolation" in this context. The input-output methods described in the paper, i.e. decreasing the weight of an edge in the network, closely resembles the site percolation methods used in network science to study the resilience of certain networks to shocks, see Chapter 15 in Newman (2010). A similar method, known as the hypothetical extraction method, is used in traditional input-output analysis to study the importance of individual sectors, but this method tends to focus on removal of vertices rather than edges (Schultz 1977; Miller and Blair 2009).

in production. The term ω^*_{ei} measures the amount of energy embodied in i's output, while $\mathbf{M}_{ij}$ measures how much j relies on i's inputs for production, both directly and indirectly. Hence, the quantity δ_{ij} is the amount of energy indirectly consumed by sector j via their direct and indirect network relationships with sector i. Moreover, δ_{ij} governs how j's consumer centrality adjusts from the energy efficiency shock. In summary, energy efficiency shocks adjust a sector's network position, and downstream-energy centrality quantifies the magnitude of this adjustment. Thus, the new topology of the network will shift toward industries with high downstream-energy centralities.

The intuition for upstream-energy centrality follows from how supplier centrality $\mathbf{Y}_k$ changes after the energy efficiency shock. The percolation of the shock is shown in sectors upstream (to the left) of the source sector in Figure 1. Using the results of Proposition 3, we can write this adjustment as

$$\frac{\partial \mathbf{Y}_k}{\partial \phi_{ei}} \phi_{ei} = (\sigma - 1)\, v_k \omega^*_{ei} \mathbf{Y}_i$$

The quantity v_k measures the direct and indirect ways k's output is used to produce energy and is the idiosyncratic component that governs how $\mathbf{Y}_k$ responds to the energy efficiency shock. As before, the goods-based input-output parameter ω^*_{ei} measures the amount of energy embodied in i's output.[15] Thus, the quantity $v_k \omega^*_{ei}$ can be interpreted as the amount of k's output that is embedded in energy resources consumed by the source sector. As the energy efficiency improvement adjusts ω^*_{ei}, the shock "percolates" to upstream producers and changes their supplier centrality by an amount proportional to v_k, their implicit position as a supplier of energy resources. Upstream-energy centrality measures the magnitude by which this implicit position adjusts in response to the energy efficiency improvement, and thus the change in network topology will favor industries with high upstream-energy centrality.

Up to this point, we have illustrated how the input-output network propagates energy efficiency shocks and how this propagation impacts the topology of the input-output net-

[15] A dual interpretation is that ω^*_{ei} measures the amount of energy necessary to produce one unit of i's output. Since $\mathbf{Y}_i$ accounts for the amount of i's output embedded in final good consumption, the quantity $\omega^*_{ei} \mathbf{Y}_i$ measures the amount of energy embodied in i's output that reaches the representative household.

work. With these results, we are now in a position to unravel the mechanics of the general equilibrium rebound effect. To relay these mechanics, we restrict our attention to how aggregate energy use changes following the energy efficiency improvement. In particular, we show that general equilibrium energy savings $\mathcal{S}_{GE} = -\frac{dy_e}{d\phi_{ei}}\phi_{ei}$ are determined through changes in the market price of the energy commodity (*price effect*), value added (*scale effect*), or the structural importance of energy in the economy (*composition effect*). The following proposition summarizes the results of this decomposition.

Proposition 4. (General Equilibrium Energy Savings) *The total change in aggregate energy use following an energy efficiency improvement in the source sector is decomposed into*

1. *A price effect given by*
$$\mathcal{S}_{price} = \sigma\left(\lambda_{ei} - \delta_{ie}\right) y_e \tag{19}$$

2. *A scale effect given by*
$$\mathcal{S}_{scale} = -\lambda_{ei} y_e \tag{20}$$

3. *A composition effect given by*
$$\mathcal{S}_{comp} = (1-\sigma)\, v_e x_{ei} \tag{21}$$

where the term $\lambda_{ei} = \frac{p_e x_{ei}}{C}$ *is a Domar weight measuring the sales share of the energy intermediate used by the source sector. Total energy savings from the energy efficiency improvement is given by combining these effects as follows*

$$\mathcal{S}_{GE} = \mathcal{S}_{price} + \mathcal{S}_{scale} + \mathcal{S}_{comp}$$

Proposition 4 links the change in aggregate energy consumption with the topology of the goods-based input-output network, captured by the energy centralities δ_{ie} and v_e. In the next few pages, we explain the economic intuition one channel at a time. We begin the discussion with the price effect.

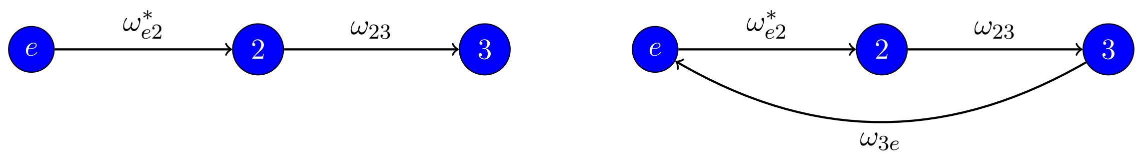
4.1 The Price Effect

The price effect is the portion of general equilibrium energy savings created by a change in the equilibrium energy price, while holding household income and composition of the economy constant. Energy savings via the price effect is created by two counteracting mechanisms. The first mechanism is what we refer to as an *input cost effect*, which we show reduces energy savings. The input cost effect is mitigated by a second mechanism, a *value added effect*, that increases energy savings. As a result, the price effect can either increase or decrease the general equilibrium energy savings, depending on which mechanism dominates.

4.1.1 The Input Cost Effect

The input cost effect is measured by the term $\sigma\delta_{ie}$ in equation (19) and, all else constant, increases energy consumption following the efficiency shock through a reduction in the price of energy. The input cost effect is proportional to δ_{ie}, the energy sector's downstream-energy centrality. To illustrate how the mechanics for reaching a lower energy price are tied up with the network of input-output linkages, consider the two stylized input-output networks illustrated in Figure 2. The two networks differ in only a single relationship between sectors—namely in Panel 2b the energy sector purchases intermediate inputs from sector 3—but this minor difference is sufficient to generate different energy savings across these two economies. In these examples, we assume sector 2 experiences an energy efficiency improvement. Holding wages constant, the energy efficiency improvement has the effect of reducing the marginal cost for producers in sector 2. Under marginal cost pricing, the reduced cost is passed on to downstream sectors, in this case sector 3, in the form of lower input prices. With lower input prices, producers in downstream sectors also experience a decline in their marginal cost, and subsequently lower output prices.

Differences between input-output networks play an important role. Because there is not a network cycle between the energy sector and sector 2, we have $\delta_{ie} = 0$ and the input costs for the energy sector are unaffected by the energy efficiency shock. In Panel 2a, the



(a) No cycles **(b)** One cycle, $e \to 2 \to 3 \to e$

Figure 2: The Input Cost Effect with Two Network Structures

topology of the network creates a barrier for the transmission of the negative price shock to the energy sector. In contrast, when a cycle exists, as in Panel 2b, we have $\delta_{ie} > 0$ and the negative price shock eventually reaches the energy sector, leading to a lower marginal cost of energy, and therefore a lower energy price. This, in turn, leads to an increase in energy consumption. The model dictates the topology of the input-output network, as captured by the downstream-energy centrality, δ_{ie}, is crucial for understanding how energy efficiency shocks impact the energy sector's input costs.

4.1.2 The Value Added Effect

The value added effect, given by the term $\sigma\lambda_{ei}$ in equation (19), relates an increase in the prevailing wage rate to fluctuations in the energy price. The mechanics of the value added effect mimic the classic aggregation procedures first proposed in Domar (1961), and later extended by Hulten (1978), for connecting aggregate value added growth with industry productivity shocks. In our model, competitive factor markets and free mobility of factor inputs implies $w_i = w$ for all industries. As a result, the economy's wage rate w is an equivalent measure for the aggregate productivity $w = \frac{GDP}{\bar{L}}$ of the economy, and fluctuations in w corresponds to fluctuations in aggregate productivity.

Lemma 1. (Reallocation, Aggregate Productivity, and Hulten's Theorem) *The energy efficiency shock induces a process of input reallocation across industries. Given sector j's share of total employment, $\theta_j = L_j/\bar{L}$, the energy efficiency shock leads to a reallocation of labor inputs governed by*

$$\frac{\partial \theta_j}{\partial \phi_{ei}}\phi_{ei} = \theta_j \left(1 - \sigma\right) \left(\lambda_{ei} - \omega^*_{ei} v_j\right) \tag{22}$$

Furthermore, given sector j's output per worker, $h_j = y_j / L_j$, and final consumption share, $\varepsilon_j = \frac{p_j c_j}{C}$, the change in valued added can be expressed as

$$\frac{d\log(C)}{d\log(\phi_{ei})} = \frac{1}{\sigma}\sum_j \varepsilon_j \frac{d\log(h_j)}{d\log(\phi_{ei})} = \lambda_{ei} \tag{23}$$

which is Hulten's (1978) theorem.

Lemma 1 states that input reallocation induced by the energy efficiency shock increases value added in the economy. Equation (22) shows how labor resources are redistributed across sectors following the energy efficiency improvement. Labor reallocation is governed by a sector's employment share θ_j and their implicit position as a supplier of energy inputs, v_j. Supposing for the moment that labor is uniformly distributed across sectors, i.e. $\theta = \theta_j = 1/N$, we can isolate the impacts of the input-output network on labor reallocation. If $\sigma > 1$, implying producers have scope for substituting labor with intermediates, then the full employment condition requires $\sum_j \frac{\partial \theta_j}{\partial \phi_{ei}} \phi_{ei} = 0$ and, therefore, employment shares will increase when $v_j > \frac{1}{N}\sum_j v_j$.

Coupling this result with the implications for aggregate productivity outlined in (23), we can flesh out the intuition behind the value added effect. When $\sigma > 1$, output per worker h_j is increasing in $\frac{\partial \boldsymbol{\Delta}_j}{\partial \phi_{ei}}$, which itself is an increasing function of δ_{ij}. The intuition is that downstream industries can take advantage of lower priced intermediate inputs, via the input cost effect, and substitute labor with cheaper intermediates. Lower input prices imply output prices decline in these industries and are subsequently passed on to other downstream industries, driving a cascading expansionary process in downstream sectors. This cascade of input reallocation raises output per worker h_j for all sectors downstream from the source sector.

Higher output levels in downstream sectors from the source sector increase demand for intermediates from upstream industries, and thus upstream industries must also expand production. Upstream industries can increase output by taking advantage of slackness in the labor market brought on by substitution in downstream industries. As labor moves from downstream industries, where intermediates are cheaper, to upstream industries facing burgeoning demand, wage rates must rise to signal relative scarcity in

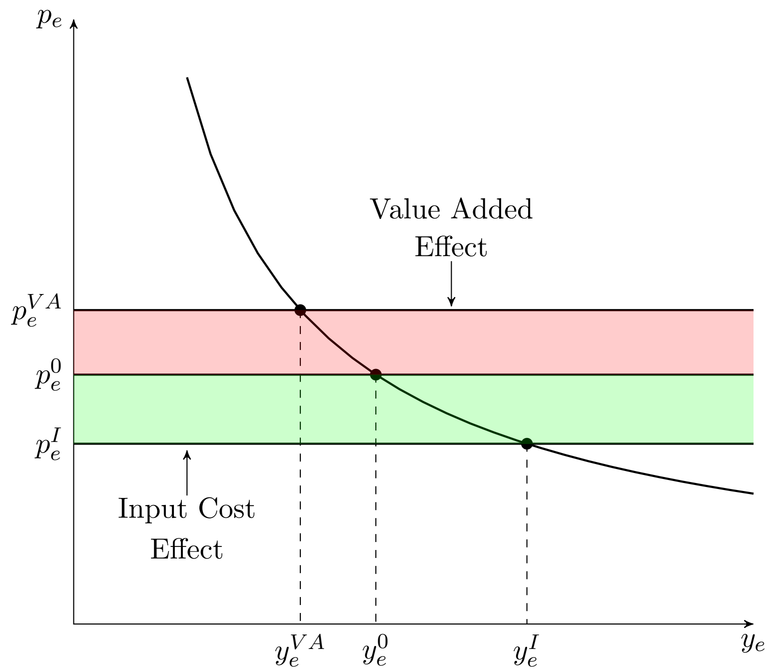


Figure 3: The Price Effect

upstream industries. The consequence of this adjustment is an increase in the prevailing wage rate in the economy, which hits prices in each industry, including the energy sector. At the conclusion of the adjustment process, higher labor costs translate into a higher price for energy, which reduces aggregate energy consumption overall.

Figure 3 illustrates the net impact of the price effect on aggregate energy use. As described above, the input cost effect p_e^I reduces aggregate energy use when $\delta_{ie} > 0$, implying the energy sector is exposed to the negative price shock originating in the source sector. As a consequence, the energy price declines and aggregate energy consumption rises from the baseline level y_e^0. The figure also shows the impact of the value added effect on energy use. Higher labor costs imply an increase in the energy price from p_e^0 to p_e^{VA}. Exposed to a higher energy price, both intermediate purchasers and consumers reduce energy consumption, driving down aggregate energy use. The input cost effect is offset by the value added effect, and the net change in aggregate energy use will depend on which effect dominates.

4.2 The Scale Effect

The scale effect corresponds to the increase in energy use (negative energy savings) driven by an expansion in household income, holding commodity prices and composition of the economy constant. The scale effect is similar to the value added effect, but different forces operate. Qualitatively, the scale effect reflects a change in the characteristics of the demand-side of the economy, whereas the value added effect focuses on the supply-side characteristics. Quantitatively, unlike the value added effect, the scale effect does not depend on the elasticity of substitution since relative prices are held constant.

From Proposition 4, we immediately discern energy savings via the scale effect are negative and energy use unambiguously increases. Consider the resource constraint of the economy given by $\bar{L} = \sum_j L_j$. We depict this relation graphically in Figure 4, using only two industries for clarity. Holding commodity prices constant at their baseline values p_i^0 and p_e^0, the energy efficiency shock increases consumer income through a shift in aggregate productivity of the economy. Because commodity prices are held constant in this scenario, an increase in household income is represented as a vertical shift in the production frontier, and the magnitude of this shift corresponds to the increase in the economy's prevailing wage rate. As a result, gross output in all industries increases, represented as a movement from point A to point B. Because output prices are held constant at their baseline values, the increase in gross output corresponds to an increase in the volume of production in each industry, including the energy sector. As gross output in the energy sector rises above the baseline value, i.e. $p_e^0 \left(y_e^1 - y_e^0\right) > 0$, the increase in the scale of the economy results in negative energy savings from the energy efficiency improvement.

4.3 The Composition Effect

The composition effect reflects how energy use responds to structural change in the economy. Holding factor and commodity prices and household income constant, the energy efficiency improvement induces a transformation in the underlying production structure of the economy. In our model, this transformation is accounted for by changes in the

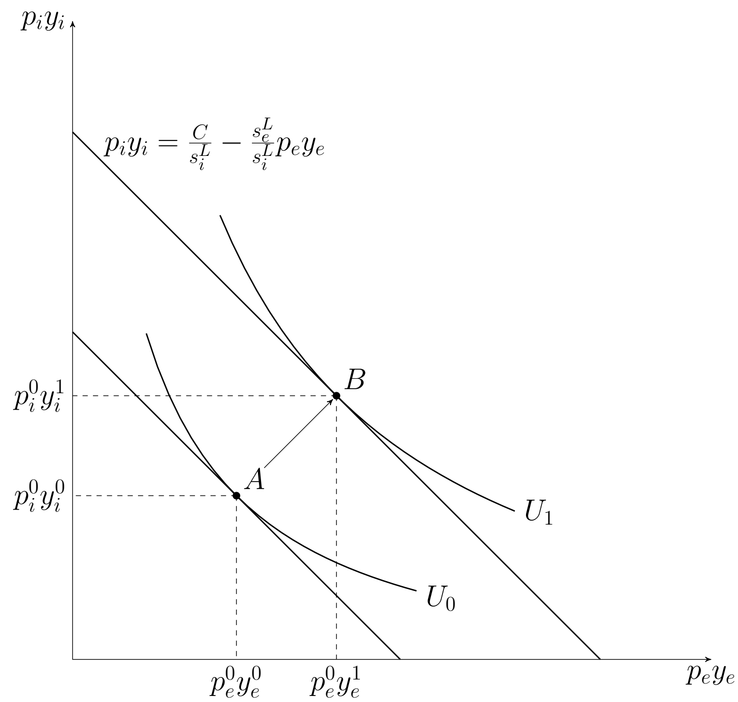


Figure 4: The Scale Effect

topology of the economy's *goods-based* input-output network. As we show below, structural transformation originates from the interaction between partial equilibrium adjustments and the economy's goods-based input-output network.

The general equilibrium energy savings from the composition effect are given in equation (21). Since the elasticity of substitution dictates whether energy savings are positive or negative, the above expression suggests input substitution plays an important role in the composition effect. The mechanics behind the composition effect, in fact, start with the standard partial equilibrium result that relies on input substitution between energy and other inputs in the source sector. The standard partial equilibrium result assumes there are no input-output linkages in the economy.

Lemma 2. (Partial Equilibrium Energy Savings without Input-Output Linkages) *In a standard partial equilibrium setting, where factor and commodity prices are held constant and input-output linkages are omitted from the model, energy efficiency improvements impact energy*

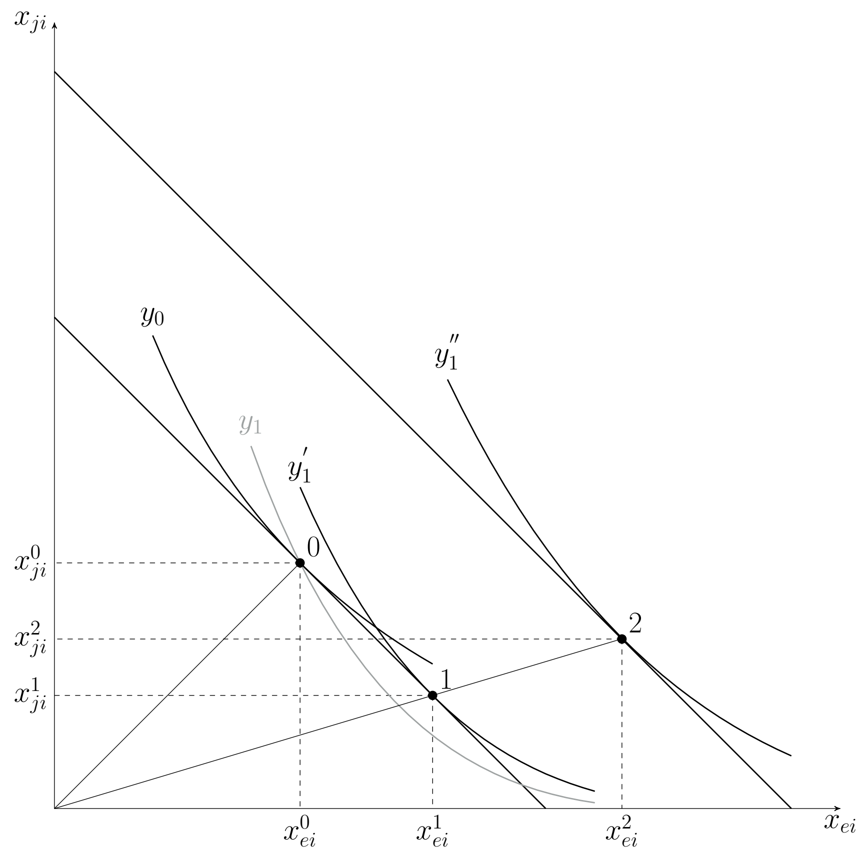


Figure 5: Partial Equilibrium and the Composition Effect ($\sigma > 1$)

consumption through a technique and energy service price effect.

$$\frac{\partial x_{ei}}{\partial \phi_{ei}}\phi_{ei} = \underbrace{-x_{ei}}_{\text{Technique Effect}} + \underbrace{\sigma x_{ei}}_{\text{Energy Service Price Effect}}$$

Therefore, partial equilibrium energy savings are expressed as

$$\mathcal{S}_{partial} = (1 - \sigma)\, x_{ei} \tag{24}$$

We depict this input substitution process graphically for $\sigma > 1$ in Figure 5. We only use two inputs for clarity, and we note the figure is defined over the space of physical units of input, i.e. a goods-based production technology. The initial isoquant is depicted as y_0, and the initial input mix is given by Point 0. The energy efficiency improvement has the effect of changing the technical rate of substitution (TRS) between energy and other inputs for producers in the source sector. In the case we show in the figure, the slope of the isoquant becomes steeper. Holding the input mix constant at the initial values, the change in the TRS is represented as a movement from y_0 to the isoquant labelled y_1. At the initial input bundle, producers in the source sector can produce y_1 for the same cost as producing y_0. However, source sector producers reallocate their input selection from Point 0 to Point 1, moving to a higher isoquant labelled $y_1^{'}$, producing more output for the same cost, and thus increasing profits in the sector. When $\sigma > 1$, the figure illustrates how producers in the source sector reallocate input selection to favor the energy intermediate. As a consequence, energy use increases from x_{ei}^0 to x_{ei}^1 and consumption of the non-energy intermediate declines from x_{ji}^0 to x_{ji}^1. This is the partial equilibrium effect given in Lemma 2.

This partial equilibrium adjustment, however, is only a first-order effect. Because sectors are connected through the network of input-output linkages, the partial equilibrium adjustment will impact output in *any* sector that implicitly produces energy resources. If the source sector also provides inputs, either directly or indirectly, to the energy sector, i.e. $v_i > 0$, then the source sector's supplier centrality will adjust following the energy efficiency shock.[16] As illustrated above, when $\sigma > 1$, the source sector will increase consumption of the energy intermediate via the partial equilibrium channel, which increases output in the energy sector. As production expands, energy producers will require more inputs to production, and when these inputs are produced using energy, the composition effect drives up energy production even further. For example, in Figure 5, the impact on the source sector's supplier centrality from the composition effect stimulates a movement from Point 1 to Point 2, increasing consumption of x_{ei}, leading to more energy use than the standard partial equilibrium result would suggest. The following Lemma summarizes this process formally.

Lemma 3. (Partial Equilibrium with Input-Output Linkages) *When sectors are connected through a network of input-output linkages, the partial equilibrium effect of energy efficiency gains in the source sector can be decomposed into a technique, energy service price, and a composition*

[16]We note that this condition implies that producers in the energy sector use non-energy intermediates in production. In the absence of such linkages, the partial equilibrium result holds.

effect

$$\frac{\partial x_{ei}}{\partial \phi_{ei}}\phi_{ei} = -\mathcal{S}_{partial} + \underbrace{x_{ei}\left(\sigma - 1\right)\omega^*_{ei}v_i}_{\text{Structural Adjustment Effect}}$$

The partial equilibrium structural adjustment in energy use induces a structural transformation in the economy where upstream suppliers of the energy sector become more central in the economy's input-output network. Formally, the change in supplier centralities following the energy efficiency shock

$$\frac{\partial \mathbf{Y}}{\partial \phi_{ei}}\phi_{ei} = \mathbf{M}\frac{\partial \mathbf{\Omega}^*}{\partial \phi_{ei}}\mathbf{Y}$$

where $\frac{\partial \mathbf{\Omega}^*}{\partial \phi_{ei}}\mathbf{Y}$ *is the partial equilibrium impact of the energy efficiency shock on the topology of the input-output network. Aggregate energy savings from the the composition effect is given by*

$$\mathcal{S}_{comp} = v_e \mathcal{S}_{partial} \tag{25}$$

The implications of Lemma 3 are that the implicit suppliers of energy services within the economy expand output levels when $\sigma > 1$ following the energy efficiency shock, all while holding commodity and factor prices constant. This composition effect is, therefore, purely a consequence of the interactions between sectors summarized by the network of input-output linkages. When sectors interact within a networked setting, general equilibrium savings, and thus rebound, will diverge from predictions made with partial equilibrium analyses. In particular, the existence of input-output linkages between sectors will create a multiplier effect on partial equilibrium savings, thereby creating variation in estimated general equilibrium rebound effects. The next proposition states this result more formally.

Proposition 5. (Multiplier Effect) *The network of input-output linkages creates a multiplier effect on partial equilibrium energy savings under the following conditions*[17]

1. *If* $\sigma < 1$*, then* $\mathcal{S}_{comp} > \mathcal{S}_{partial} > 0$
2. *If* $\sigma > 1$*, then* $\mathcal{S}_{comp} < \mathcal{S}_{partial} < 0$

[17]From the power series expansion of $\mathbf{M}_{ee} = \left(1 + \mathbf{\Omega}^*_{ee} + \mathbf{\Omega}^{*2}_{ee} + ..\right)$, it is clear that $\mathbf{M}_{ee} < 1$ only occurs if some entries in $\mathbf{\Omega}^*$ are negative. Since this is not the case, we can rule out the possibility that $\mathbf{M}_{ee} < 1$.

Proposition 5 shows how the network of input-output linkages translates partial equilibrium adjustments from energy efficiency improvements into aggregate energy savings. More telling, Proposition 5 shows a general equilibrium setting is not even required for input-output linkages to matter. The simple consideration of input-output linkages, holding prices and incomes constant, still creates substantial variation in partial equilibrium energy savings, and thus the partial equilibrium rebound effect. With this final result, we turn to how the topology of the input-output network determines the general equilibrium rebound effect.

4.4 General Equilibrium Rebound Effect

The main insight of this paper is that the topology of the input-output network will impact estimates of the general equilibrium rebound effect. The previous sections laid the groundwork for understanding the mechanics that drive general equilibrium rebound as well as providing the necessary mathematical foundations. In the following proposition, we combine each of the channels discussed above into an expression for the general equilibrium rebound effect.

Proposition 6. (General Equilibrium Rebound Effect) *The general equilibrium rebound effect is defined as*

$$\mathcal{R}_{GE} = \frac{\mathcal{S}_{pot} - \mathcal{S}_{GE}}{\mathcal{S}_{pot}}$$

where $\mathcal{S}_{pot}$ is the potential (or, engineering) estimate of energy savings. The potential energy savings is given by the technique effect in Lemma 2. Combining the results from Proposition 4 into the definition of general equilibrium rebound, the general equilibrium effect can be decomposed into network and non-network components

$$\mathcal{R}_{GE} = 1 + \underbrace{(\sigma - 1)\, v_e + \sigma \frac{\delta_{ie}}{s_{ei}}}_{\substack{\text{Network} \\ \text{Component}}} + \underbrace{(1 - \sigma)\, \lambda_e}_{\substack{\text{Non-network} \\ \text{Component}}}$$

where $s_{ei} = \frac{x_{ei}}{y_e}$ is the source sector's share of energy consumption and λ_e is the energy sector's Domar weight.

Proposition 6 states the magnitude of the general equilibrium effect, in theory, is affected by topological details of the economy's input-output network, namely the energy centralities v_e and δ_{ie}. The proposition also establishes how other features of the economy might impact estimates of general equilibrium rebound, such as the elasticity of substitution σ and source sector's share of aggregate energy use s_{ei}.[18]

Before turning to the simulation, we introduce a final theoretical result that helps explain the possibility of a negative rebound effect, typically referred to as "super conservation" (Saunders 2008). Super conservation occurs when actual energy savings exceed the potential energy savings from the energy efficiency improvement, hence rebound is negative. Proposition 5 suggests energy savings from the composition effect can exceeds the partial equilibrium prediction when $\sigma < 1$, raising the possibility of a negative rebound effect from the composition channel.[19] The following corollary illustrates how the topological details of the input-output network play a role in super conservation.

Corollary 1. (Composition and Super Conservation) *Following the results in Proposition 5, the composition effect leads to super conservation when $\sigma < 1$ and*

$$v_e > \frac{1}{1-\sigma} \tag{26}$$

The results of the corollary suggest two features of numerical models may explain negative rebound effects. First, all else constant, small values of the elasticity of substitution are more likely to generate negative rebound effects. Since $v_e > 1$, lower values for σ make the inequality more likely to be satisfied. As $\sigma \to 0$, i.e. approaches the Leontief limit, super conservation from the composition channel is guaranteed to occur in the pres-

[18]The energy intensiveness of the source sector is included in the downstream-energy centrality, δ_{ie} since ω^*_{ei} is proportional to this value. Based on the expression for general equilibrium rebound, we find the rebound effect unambiguously increases when the source sector is more energy intensive. This finding, for instance, aligns with the results found in Saunders (2008).

[19]Negative rebound can also theoretically occur from the price effect. The condition for super conservation to occur from the price channel is when

$$\lambda_e > \frac{1}{\sigma} + \frac{\delta_{ie}}{s_{ei}}$$

When $\sigma < 1$, this inequality is unlikely to be satisfied in practice since this implies energy sales would need to be substantially larger than overall GDP. When $\sigma > 1$, the inequality could be satisfied but would require the value added effect to be substantially larger than the input cost effect.

ence of input-output linkages. Second, if the energy sector's downstream-energy centrality v_e is sufficiently large, then super conservation can occur. This implies the opportunity for energy savings from energy efficiency improvements is larger when the energy sector directly or indirectly relies on the use of energy for production. This result raises an important conceptual distinction between the energy sector and the energy system more broadly. With input-output linkages, the production of energy resources may also rely on intermediate goods with some amount of embodied energy; in this sense, the network of input-output linkages both structures and delineates the boundaries of the economy's energy system, which consists of energy resources, technologies, and uses (Blackburn et al. 2017). The upstream-energy centrality v_e of the energy sector reflects the importance of considering the energy system as a whole because the boundaries of the energy system are no longer constrained to only downstream technologies and uses. With this final theoretical result, we next take to the model to data to evaluate the predictions of our theory.

5 Application

In this section, we simulate the rebound effect from exogenous, industrial energy efficiency shocks. We collect proprietary, input-output data for each US state in 2015 from the IMPLAN Group (IMPLAN). The IMPLAN datasets cover more than 500 industries for each state, providing a rich disaggregation to investigate the impact of input-output networks on the general equilibrium rebound effect. After calibrating the model, we apply successive energy efficiency shocks to the input-output relationship between the energy sector and other sectors in the economy. We consider energy efficiency improvements that affect the input-output coefficients with respect to three energy supplying industries: (i) Coal Mining (NAICS 212111-212113), (ii) Petroleum Refineries (NAICS 324110), and (iii) Natural gas distribution (NAICS 221210).

Our calibration strategy generally follows the calibration in Baqaee (2018), and the precise details of the calibration procedure are outlined in Appendix B. Calibrating the model to the IMPLAN data requires the assumption that the data is in a pre-shock equilibrium. Under this assumption, factor and commodity prices are equal to their pre-shock equilibrium values, so that $w = 1$ and $\mathbf{P} = \mathbf{1}$. When this holds, all model parameters can be calibrated using the IMPLAN data. We also set $\bar{L} = 1$ for each state to eliminate the impact of differences in labor endowments to simulated rebound effects.

We simulate the rebound effect by iteratively applying a 10% energy efficiency improvement to each sector covered by the IMPLAN data.[20] For each state and energy sector, there are potentially 526 simulations, since there are 526 sectors. To address the role of the elasticity of substitution, we conduct the simulations using 6 different values for σ. However, we note that the available literature points to $\sigma < 1$ as the appropriate approximation for the elasticity of substitution between value added and intermediate inputs in production. Atalay (2017), for instance, estimates $\sigma \in [0.4, 0.8]$, while Böhringer and Rivers (2018) note the elasticity of substitution between energy and value added is approximately 0.5. With additional simulations conducted for each value of σ, the total potential simulations are $526 \times 50 \times 6 = 157,800$ for each energy sector. When a sector does not utilize energy as an intermediate input, we skip over this sector. Variation in the number of simulations presented in Table 1 shows how different energy sectors are more or less pervasive across different industrial sectors and states.

5.1 Main Results

We report the summary statistics from the simulations in Table 1. For each energy sector and value of σ, we report the mean, standard deviation, minimum, and maximum values to provide a rough indication for how the rebound effect is distributed within each simulation. We begin our discussion by noting our simulations reproduce the two empirical regularities found in the literature. Namely, these regularities are (i) the general equilibrium effect can be substantially higher than the partial equilibrium prediction, and (ii) estimates of the general equilibrium effect are highly varied. On the first empirical regularity, the simulations show that, while the average rebound effect roughly accords with

[20] The terminology ”simulation” is intended to mean that we are computing equilibrium output following the introduction of the energy efficiency shocks. The closed-form solution to the model allows us to compute output directly from the set of first-order conditions of the model. We do not need to utilize a solution algorithm for this purpose.

the partial equilibrium prediction, individual estimates of the rebound effect can be larger than twice the partial equilibrium value. On the second empirical regularity, we note the results in Table 1 are highly varied. In each case, the minimum estimated rebound effect falls below the partial equilibrium prediction, while the largest rebound effect is approximately double. However, unlike some estimates in the literature, we do not find evidence of a negative rebound effect.[21]

Table 1: Summary Statistics for Simulated Rebound Effects

	Mean	Std	Min	Max	N
Coal Mining					
$\sigma = 0.25$	0.2875	0.0291	0.2171	0.5397	8,002
$\sigma = 0.5$	0.5233	0.0376	0.4719	1.0109	8,002
$\sigma = 0.75$	0.7647	0.0537	0.7328	1.5468	8,002
$\sigma = 1.25$	1.2652	0.0938	1.2413	2.6743	8,002
$\sigma = 1.5$	1.5245	0.1158	1.4885	3.2695	8,002
$\sigma = 1.75$	1.7900	0.1387	1.7416	3.8856	8,002
Natural Gas Distribution					
$\sigma = 0.25$	0.3121	0.0115	0.3083	0.5435	21,488
$\sigma = 0.5$	0.5378	0.0232	0.5334	1.0035	21,488
$\sigma = 0.75$	0.7689	0.0352	0.7639	1.4768	21,488
$\sigma = 1.25$	1.2480	0.0601	1.2402	2.4579	21,488
$\sigma = 1.5$	1.4962	0.0729	1.4861	2.9661	21,488
$\sigma = 1.75$	1.7504	0.0861	1.7380	3.4865	21,488
Petroleum Refining					
$\sigma = 0.25$	0.2968	0.0244	0.2253	0.5455	21,078
$\sigma = 0.5$	0.5309	0.0359	0.4774	1.0186	21,078
$\sigma = 0.75$	0.7706	0.0529	0.7356	1.5540	21,078
$\sigma = 1.25$	1.2675	0.0923	1.2412	2.6849	21,078
$\sigma = 1.5$	1.5250	0.1134	1.4882	3.2793	21,078
$\sigma = 1.75$	1.7887	0.1353	1.7412	3.8940	21,078

The variation across rows shows how even minor differences in assumptions for σ can

[21] Given the results of Corollary 1, we could set the elasticity of substitution to a sufficiently low value to generate a negative rebound effect from the composition channel. In our data, the maximum of v_e is 1.15, which implies $\sigma < 0.13$ would be sufficient to generate negative rebound from the composition channel. However, rebound from the composition channel would need to be larger than the price and scale effect to result in an overall negative general equilibrium rebound effect.

generate sizable differences in the rebound effect. As an example, suppose two different applications assumed $\sigma = 0.5$ and $\sigma = 0.75$, respectively. Both values could plausibly be used based on available evidence. Our results suggest that even this minor difference in modeling choice would increase the variation in available estimates quite substantially. Specifically, the difference in choices leads to a scenario where the maximum estimate becomes more than three times larger than the minimum. This minor difference in assumptions creates substantial uncertainty regarding the actual size of the general equilibrium rebound effect.

Our results also suggest that even reasonable assumptions regarding the elasticity of substitution can generate backfire. Recall, backfire refers to a situation where the energy efficiency improvement causes aggregate energy use to increase. For each energy sector, we find the minimum elasticity of substitution required to generate backfire is 0.5, which accords with the value used in Böhringer and Rivers (2018). This finding suggests we don't need large values for σ to have energy efficiency investments backfire. Notably, once we move into flexible substitution, i.e. $\sigma > 1$, backfire is nearly guaranteed. However, the available estimates for σ suggest intermediate inputs and value added are complements rather than substitutes.

Variation within each row shows how different assumptions regarding the sector experiencing the energy efficiency improvement impacts estimated rebound effects. In the next section, we explore how the different elements of the model contribute to variation in estimates for each value of the elasticity of substitution.

5.2 Explaining Variation in Simulated Rebound

Table 1 shows variation in simulated rebound effects along two dimensions. The first dimension is relative to choices made about the elasticity of substitution in the simulation. We discussed in the previous section how this can contribute to variation in simulated general equilibrium rebound. Intuitively, this result can be thought of how different choices in modeling frameworks, via different assumptions about σ, contribute to observed variation in estimates. Our results also show rebound effect varies along a second dimension within a particular model framework characterized by the same elasticity of substitution. Our results suggest this variation can still be quite substantial since the maximum values for rebound are approximately two times larger than the minimum values within each row. In this section, we show using our theoretical predictions from Proposition 6 why variation might exist even under a common modeling framework.

Proposition 6 suggests the general equilibrium rebound effect is composed of network and non-network components. The network component corresponds to the portion of the rebound effect that is attributable to variation in the energy centralities v_e and δ_{ie}, whereas the non-network components are the source sector's share of energy use s_{ei}, the source sector's energy intensity ω^*_{ei}, and λ_e the energy sector's Domar weight.[22] Importantly, holding the elasticity of substitution constant, Proposition 6 implies the general equilibrium rebound effect can be written as a linear combination of these components

$$\mathcal{R}_{GE} = \beta_0 + \beta_1 v_e + \beta_2 \frac{\delta_{ie}}{s_{ei}} + \beta_3 \lambda_e$$

The expression above allows us to utilize standard regression techniques to isolate the impacts of both the network and non-network components on variation in simulated rebound effects. To isolate the impact of the network component, we proceed as follows. First, we regress v_e and δ_{ie} on the non-network components of the model to obtain variation in the network component that is orthogonal to the non-network components. Second, we regress the simulated rebound effect on this orthogonal component while holding the non-network components constant at their average values. Third, we compute the adjusted-R^2 from the estimation to determine the fraction of variation in the general equilibrium rebound effect explained by the network component of the model. Table 2 summarizes the results of this procedure.

Table 2 reports the fraction of variation in simulated rebound explained by the network components of the model. We report the results for each elasticity of substitution

[22]We elect to remove the variation from ω^*_{ei} from the downstream-energy centrality since this captures the energy intensiveness of an industry and not the topological details of the input-output network. This choice does not affect the broader interpretation of our theoretical and empirical results. Based on this, the empirical results should be interpreted in the context of holding energy intensity constant, while allowing the topology of the input-output network to vary across applications.

Table 2: Variation in Rebound Effect Explained by the Network Component

	Coal Mining	Natural Gas Distribution	Petroleum Refining
$\sigma = 0.25$	0.382	0.922	0.508
$\sigma = 0.50$	0.564	0.924	0.495
$\sigma = 0.75$	0.593	0.925	0.478
$\sigma = 1.25$	0.566	0.925	0.463
$\sigma = 1.50$	0.554	0.925	0.460
$\sigma = 1.75$	0.545	0.925	0.458

and energy sector for direct comparison with the results in Table 1. We find variation in the topology of the input-output network explains a non-trivial fraction of the variability in simulated rebound. In particular, we find variation in the network structure explains between 38-93 percent of the variation in simulated rebound effects. Since the non-network components of the model are held constant, these results have important practical implications for numerical approaches for estimating the general equilibrium rebound effect.

5.3 Networks, Simulations, and Rebound

The numerical results presented above suggest the topology of the input-output network is an important source of variation in numerical estimates of general equilibrium rebound. This finding suggests that the details of the microeconomic production structure of numerical approaches may lead to a wide range of estimates. A better understanding of how the microeconomic details of numerical models impact the estimated aggregate effects of energy efficiency could yield a more refined framework for understanding the output of numerical approaches.

Our findings suggest variation in microeconomic elasticities of substitution can explain a large amount of variation in numerical estimates of general equilibrium rebound. However, elasticities are not the only source of variation. Our results suggest variation can also be attributed to the modeler's choice of which sectors to apply an energy efficiency improvement. In particular, we find the relative size of the energy sector and the source sector's energy intensiveness and share of resource use will contribute to variation in estimated rebound effect. Based on our results, these outside features of the model could contribute up to 60 percent in overall variation across studies.

The remaining source of variation in estimated rebound effects turns to the economy's input-output network. Our results suggest that even if controlling for elasticities of substitution and a modeler's choice of sectors to study, the estimated aggregate effects of energy efficiency would still potentially exhibit substantial variation. We show this remaining variation is attributable to variation in the topology of the input-output network used to calibrate the structural details of numerical models. On a practical level, this result suggests that numerical models should be expected to generate varied estimates of general equilibrium rebound based on their construction. Moreover, on a conceptual level, since the topology of the input-output network reflects the underlying structural details of the economy, our results suggest a need to abandon the notion that partial or general equilibrium rebound converges to a unique value. Under this "unique rebound" notion, a wide range of outcomes could diminish the credibility of available estimates, particularly from numerical models, even though the sources of variation are economically meaningful.

6 Conclusion

In this paper, we set out to bring theory closer to current numerical approaches for estimating general equilibrium rebound. To this end, we pinpointed a common feature of numerical models, namely the input-output network, and implemented this feature into a general equilibrium framework with energy efficiency. Our main results suggest the topology of the input-output network has important implications for both the mechanics and magnitude of the general equilibrium rebound effect. We show the price, scale, and composition effects that arise from idiosyncratic, energy efficiency improvements are all shaped by the network of input-output linkages. By calibrating the model to data, we offer the first examination for how variation both within and across input-output networks affect estimates for the general equilibrium rebound effect.

A key feature of our approach is that we directly model energy efficiency breakthroughs within the context of an input-output network. Our main results suggest the interaction between technical efficiency gains in intermediate inputs and the economy's input-output network has important implications for the way breakthrough innovations manifest across the economy. We find a sector's position within the economy's input-output network is a critical predictor for how it responds to efficiency shocks occurring elsewhere in the economic system. Specifically, our main theoretical results show how equilibrium responses to the efficiency shock are better characterized by centrality concepts that are more consistent with a notion of absorption rather than transmission. This finding is critical since it implies equilibrium network concepts, such as Bonacich centrality, are no longer enough for studying the transmission of idiosyncratic shocks in situations where efficiency gains directly alter the structure of the input-output network.

These results suggest that some features of the input-output network shape the distribution of surplus from technical efficiency gains. Our model predicts that output responds more in sectors that are more exposed to the shock based on their network position. When these network positions are uniformly distributed, the topology of the input-output network is more pliant, and the surplus of the efficiency gain is also uniformly distributed across sectors. However, in the case of non-uniformity, the topology of the network is more rigid, and most of the surplus from efficiency gains is accumulated in very few sectors.

Our results are illustrative of a critical conceptual distinction in models with input-output networks. That is, our findings suggest alterations in the network topology of production can lead to non-trivial changes in the architecture of the input-output network. In the context of energy efficiency, we show how changes in the input-output network's topology, coupled with other microeconomic details, can lead to an input-output architecture where more energy resources are embodied in economic output, and as a consequence, existing energy systems become more entrenched within the economic system. This finding has important implications for resource management strategies since it suggests that modern energy systems are a complex nexus of goods and services that extend far beyond the traditional schematic of energy resources, technologies, and uses.

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Appendix A Proofs

Proof of Proposition 1. We can solve for the equilibrium prices and quantities as follows. The solution for equilibrium prices follow directly from marginal cost pricing. Under this assumption, we have a $N \times N$ system of equations defined by

$$p_i^{1-\sigma} = \gamma_i w^{1-\sigma} + \omega_{ei}^* p_e^{1-\sigma} + \sum_{j \neq e} \omega_{ji} p_j^{1-\sigma}$$

Converting this system into matrix notation, and solving the system for the $N \times 1$ price vector $\mathbf{P}$ yields the solution. For the equilibrium "sales" vector, we begin with the market clearing condition for an industry given by

$$y_i = c_i + \sum_j x_{ij}$$

Substituting the expressions for final and intermediate demand yields the system

$$p_i^{\sigma} y_i = \alpha_i C + \sum_j \omega_{ij}^* p_j^{\sigma} y_j$$

Converting this expression to matrix notation and solving yields the solution for the equilibrium "sales" vector. Lastly, to solve for equilibrium wage rates, start with the labor market clearing condition given by

$$\bar{L} = \sum_j L_j$$

Substituting conditional labor demand and multiplying both sides by w yields the income constraint of the economy given by

$$C = w^{1-\sigma} \sum_i \gamma_i \mathbf{Y}_i C$$

Solving for the economy's wage rate yields

$$w = \left(\sum_i \gamma_i \mathbf{Y}_i \right)^{\frac{1}{\sigma-1}}$$

which is the solution given in Proposition 1.

■

Derivation of Definition 2. An expression for the input-output coefficient can be derived by re-arranging equation (6a). Formally, we find the exogenous input-output coefficient for energy ω_{ei} is expressed in energy-service units. That is,

$$\begin{aligned}\omega_{ei} &= \phi_{ei}^{1-\sigma} \frac{p_e^{\sigma} x_{ei}}{p_i^{\sigma} y_i} \\ &= \frac{p_{si}^{\sigma} s_{ei}}{p_i^{\sigma} y_i}\end{aligned}$$

where s_{ei} is the energy-service and p_{si} is the energy-service price. By multiplying ω_{ei} by $\phi_{ei}^{\sigma-1}$, we adjust the input-output coefficient by the productivity of the conversion technology. This implies the adjusted input-output coefficient is expressed as ratios of physical units. In other words,

$$\omega_{ei}^{*} = \phi_{ei}^{\sigma-1} \omega_{ei} = \frac{p_e^{\sigma} x_{ei}}{p_i^{\sigma} y_i}$$

■

Proof of Proposition 2. With a change of parameters following Definition 2, the services-based CES production function defined in energy services, denoted as y_i^S, is equivalent to a goods-based CES production function defined in physical units of energy, denoted as

y_i^G. The equivalence is straightforward to illustrate.

$$\begin{aligned}
\omega_{ei}^{\frac{1}{\sigma}} x_{si}^{\frac{\sigma-1}{\sigma}} &= \left(\frac{p_{si}^{\sigma} x_{si}}{p_i^{\sigma} y_i}\right)^{\frac{1}{\sigma}} (\phi_{ei} x_{ei})^{\frac{\sigma-1}{\sigma}} \\
&= \left[\frac{\left(p_e/\phi_{ei}\right)^{\sigma} (\phi_{ei} x_{ei})}{p_i^{\sigma} y_i}\right]^{\frac{1}{\sigma}} (\phi_{ei} x_{ei})^{\frac{\sigma-1}{\sigma}} \\
&= \left(\frac{p_e^{\sigma} x_{ei}}{p_i^{\sigma} y_i}\right)^{\frac{1}{\sigma}} x_{ei}^{\frac{\sigma-1}{\sigma}} \\
&= (\omega_{ei}^{*})^{\frac{1}{\sigma}} x_{ei}^{\frac{\sigma-1}{\sigma}}
\end{aligned}$$

Therefore, we have illustrated that following Definition 2, we have that $y_i^S = y_i^G$. Using this equivalence, we can show how each production technology responds to the shock and these responses are equivalently expressed as

$$\begin{aligned}
\frac{\partial y_i^S}{\partial \phi_{ei}} &= \frac{\partial y_i^G}{\partial \phi_{ei}} \\
\left(\frac{\sigma-1}{\sigma}\right) \omega_{ei}^{\frac{1}{\sigma}} x_{si}^{\frac{\sigma-1}{\sigma}} \frac{\partial x_{si}}{\partial \phi_{ei}} \frac{\phi_{ei}}{x_{si}} &= \frac{1}{\sigma} (\omega_{ei}^{*})^{\frac{1}{\sigma}} x_{ei}^{\frac{\sigma-1}{\sigma}} \frac{\partial \omega_{ei}^{*}}{\partial \phi_{ei}} \frac{\phi_{ei}}{\omega_{ei}^{*}} \\
(\sigma-1) \frac{\partial x_{si}}{\partial \phi_{ei}} \frac{\phi_{ei}}{x_{si}} &= \frac{\partial \omega_{ei}^{*}}{\partial \phi_{ei}} \frac{\phi_{ei}}{\omega_{ei}^{*}}
\end{aligned}$$

■

Proof of Proposition 3. Given Assumption 1 holds, **M** can be written as a power series expansion

$$\mathbf{M} = \sum_{k=0}^{\infty} \left(\boldsymbol{\phi}^{\sigma-1} \odot \boldsymbol{\Omega}\right)^{k} = \sum_{k=0}^{\infty} \boldsymbol{\Omega}^{*k}$$

where the exponent k represents a matrix power. The k^{th} order impact of the energy efficiency improvement is related to the first-order impact through the following relationship

$$\begin{aligned}
\frac{\partial \boldsymbol{\Omega}^{*k}}{\partial \phi_{ei}} &= \frac{\partial \boldsymbol{\Omega}^{*}}{\partial \phi_{ei}} \boldsymbol{\Omega}^{*(k-1)} + \sum_{l=1}^{k-2} \boldsymbol{\Omega}^{*(k-l-1)} \frac{\partial \boldsymbol{\Omega}^{*}}{\partial \phi_{ei}} \boldsymbol{\Omega}^{*l} + \boldsymbol{\Omega}^{*(k-1)} \frac{\partial \boldsymbol{\Omega}^{*}}{\partial \phi_{ei}} \\
&= (\sigma-1)\, \omega_{ei}^{*} \left(\frac{\partial \boldsymbol{\phi}}{\partial \phi_{ei}} \boldsymbol{\Omega}^{*(k-1)} + \sum_{l=1}^{k-2} \boldsymbol{\Omega}^{*(k-l-1)} \frac{\partial \boldsymbol{\phi}}{\partial \phi_{ei}} \boldsymbol{\Omega}^{*l} + \boldsymbol{\Omega}^{*(k-1)} \frac{\partial \boldsymbol{\phi}}{\partial \phi_{ei}} \right)
\end{aligned}$$

Using this relationship, the impact of the energy efficiency shock on the economy's multiplier matrix can be written as

$$\frac{d\mathbf{M}}{d\phi_{ei}}\phi_{ei} = (\sigma - 1)\,\omega_{ei}^{*}\left(\frac{\partial \boldsymbol{\phi}}{\partial \phi_{ei}} + \boldsymbol{\Pi}_{\phi_{ei}}\right)$$

where the elements of $\boldsymbol{\Pi}_{\phi_{ei}} = \sum_{k=2}^{\infty} \frac{\partial \boldsymbol{\Omega}^{*k}}{\partial \phi_{ei}}$ are finite because the sequence $\sum_{k=2}^{\infty} \frac{\partial \boldsymbol{\Omega}^{*k}}{\partial \phi_{ei}}$ converges to 0 given Assumption 1 holds. The derivative of the Leontief inverse is given by

$$\frac{\partial \mathbf{M}}{\partial \phi_{ei}}\phi_{ei} = -\mathbf{M}\frac{\partial\left[\mathbf{I} - \boldsymbol{\Omega}^{*}\right]}{\partial \phi_{ei}}\mathbf{M}\phi_{ei} = \mathbf{M}\frac{\partial \boldsymbol{\Omega}^{*}}{\partial \phi_{ei}}\mathbf{M}\phi_{ei} = (\sigma - 1)\,\omega_{ei}^{*}\mathbf{M}\frac{\partial \boldsymbol{\phi}}{\partial \phi_{ei}}\mathbf{M}$$

Hence, we have that

$$\frac{\partial \mathbf{M}}{\partial \phi_{ei}}\phi_{ei} = (\sigma - 1)\,\omega_{ei}^{*}\mathbf{M}\frac{\partial \boldsymbol{\phi}}{\partial \phi_{ei}}\mathbf{M} = \sum_{k}^{\infty} \frac{\partial \boldsymbol{\Omega}^{*k}}{\partial \phi_{ei}}$$

■

Proof of Proposition 4: Macroeconomic Price Effect. The energy price is written as

$$p_e = \boldsymbol{\Delta}_e^{\frac{1}{1-\sigma}} w$$

Differentiating this expression with respect to the energy efficiency improvement we have

$$\begin{aligned}
\frac{\partial p_e}{\partial \phi_{ei}}\phi_{ei} &= \frac{1}{1-\sigma}\boldsymbol{\Delta}_e^{\frac{1}{1-\sigma}}\boldsymbol{\Delta}_e^{-1}\frac{\partial \boldsymbol{\Delta}_e}{\partial \phi_{ei}}\phi_{ei} w + \boldsymbol{\Delta}_e^{\frac{1}{1-\sigma}}\frac{\partial w}{\partial \phi_{ei}}\phi_{ei} \\
&= \frac{1}{1-\sigma}\boldsymbol{\Delta}_e^{\frac{1}{1-\sigma}}\boldsymbol{\Delta}_e^{-1}\left[(\sigma - 1)\,\omega_{ei}^{*}\boldsymbol{\Delta}_e \mathbf{M}_{ie}\right] w + \boldsymbol{\Delta}_e^{\frac{1}{1-\sigma}}\frac{\partial \left(\boldsymbol{\alpha}'\boldsymbol{\Delta}\right)^{\frac{1}{\sigma-1}}}{\partial \phi_{ei}}\phi_{ei} \\
&= -p_e\omega_{ei}^{*}\mathbf{M}_{ie} + \boldsymbol{\Delta}_e^{\frac{1}{1-\sigma}}\frac{1}{1-\sigma}\left(\boldsymbol{\alpha}'\boldsymbol{\Delta}\right)^{\frac{1}{\sigma-1}}\left(\boldsymbol{\alpha}'\boldsymbol{\Delta}\right)^{-1}\sum_{j}\frac{\partial \boldsymbol{\Delta}_j}{\partial \phi_{ei}}\alpha_j\phi_{ei} \\
&= -p_e\omega_{ei}^{*}\mathbf{M}_{ie} + \frac{1}{\sigma-1}p_e\left(\boldsymbol{\alpha}'\boldsymbol{\Delta}\right)^{-1}(\sigma - 1)\,\boldsymbol{\Delta}_e\omega_{ei}^{*}\sum_{j}\mathbf{M}_{ij}\alpha_j \\
&= -p_e\omega_{ei}^{*}\mathbf{M}_{ie} + p_e w^{\sigma-1}\boldsymbol{\Delta}_e\omega_{ei}^{*}\mathbf{Y}_i
\end{aligned}$$

For the second term, we can substitute the parameter values to arrive at

$$
\begin{aligned}
\frac{\partial p_e}{\partial \phi_{ei}}\phi_{ei} &= -p_e\omega_{ei}^*\mathbf{M}_{ie} + p_e w^{1-\sigma}\left(\frac{p_e}{w}\right)^{1-\sigma}\left(\frac{p_e^\sigma x_{ei}}{p_i^\sigma y_i}\right)\left(\frac{p_i^\sigma y_i}{C}\right) \\
&= -p_e\omega_{ei}^*\mathbf{M}_{ie} + p_e\left(\frac{p_e x_{ei}}{C}\right) \\
&= -p_e\omega_{ei}^*\mathbf{M}_{ie} + p_e\lambda_{ei} \\
&= p_e\left(\lambda_{ei} - \omega_{ei}^*\mathbf{M}_{ie}\right) \\
&= p_e\left(\lambda_{ei} - \delta_{ie}\right)
\end{aligned}
$$

Hence, energy savings via the price channel can be expressed as

$$
\begin{aligned}
\mathcal{S}_{price} = -\frac{dy_e}{d\phi_{ei}}\phi_{ei} = \frac{\partial p_e^{-\sigma}}{\partial \phi_{ei}}C\mathbf{Y}_e \\
&= -\sigma p_e^{-\sigma-1}\frac{\partial p_e}{\partial \phi_{ei}}\phi_{ei}C\mathbf{Y}_e \\
&= \sigma\left(\lambda_{ei} - \delta_{ie}\right)y_e
\end{aligned}
$$

■

Proof of Proposition 4: Scale Effect. Differentiating consumer income with respect to the energy efficiency improvement yields the following

$$
\begin{aligned}
\frac{\partial C}{\partial \phi_{ei}} &= \frac{\partial w\bar{L}}{\partial \phi_{ei}}\bar{L} \\
&= \bar{L}\frac{\partial\left(\boldsymbol{\alpha}'\boldsymbol{\Delta}\right)^{\frac{1}{\sigma-1}}}{\partial \phi_{ei}} \\
&= Cw^{1-\sigma}\left(\frac{p_e}{w}\right)^{1-\sigma}\left(\frac{p_e^\sigma x_{ei}}{p_i^\sigma y_i}\right)\left(\frac{p_i^\sigma y_i}{C}\right) \\
&= \lambda_{ei}C
\end{aligned}
$$

Using this result, we can express energy savings from the scale effect as

$$\begin{aligned}\mathcal{S}_{scale} &= -\frac{dy_e}{d\phi_{ei}}\phi_{ei} \\ &= \frac{\partial C}{\partial \phi_{ei}} p_e^{-\sigma}\mathbf{Y}_e \\ &= -\lambda_{ei} y_e\end{aligned}$$

■

Proof of Proposition 4: Composition Effect. The composition effect is given

$$\begin{aligned}-\frac{dy_e}{d\phi_{ei}}\phi_{ei} &= -p_e^{-\sigma} C \frac{\partial \mathbf{Y}_e}{\partial \phi_{ei}}\phi_{ei} \\ &= p_e^{-\sigma} C \left(1-\sigma\right) \omega_{ei}^{*} \mathbf{M}_{ee} \mathbf{Y}_i \\ &= \left(1-\sigma\right) \mathbf{M}_{ee} x_{ei} \\ &= \left(1-\sigma\right) v_e x_{ei}\end{aligned}$$

■

Proof of Lemma 1. The change in conditional labor demand after the energy efficiency improvement is given by

$$\begin{aligned}\frac{\partial L_j}{\partial \phi_{ei}}\phi_{ei} &= \frac{\partial \mathbf{Y}_j}{\partial \phi_{ei}}\gamma_j w^{-\sigma} C + \mathbf{Y}_j \gamma_j \frac{\partial w^{-\sigma}}{\partial \phi_{ei}} C + \mathbf{Y}_j \gamma_j w^{-\sigma}\frac{\partial C}{\partial \phi_{ei}} \\ &= \left(\sigma-1\right) \omega_{ei}^{*} \mathbf{M}_{je} L_j + \left(1-\sigma\right) \lambda_{ei} L_j \\ &= \left(\sigma-1\right) \omega_{ei}^{*} v_j \theta_j \bar{L} + \left(1-\sigma\right) \lambda_{ei} \theta_j \bar{L}\end{aligned}$$

where $\theta_j = \frac{L_j}{\bar{L}}$ is the employment share in industry j. Hence, we can write the change in a sector's employment share as

$$\frac{\partial \theta_j}{\partial \phi_{ei}}\phi_{ei} = \theta_j \left(1-\sigma\right) \left(\lambda_{ei} - \omega_{ei}^{*} v_j\right)$$

Suppose $\sigma > 1$, implying there is limited scope for substitution. Then, this would be imply labor share increases whenever

$$\frac{\lambda_{ei}}{\omega_{ei}^*} < v_j$$

The full employment condition implies that

$$\sum_j \frac{\partial \theta_j}{\partial \phi_{ei}} \phi_{ei} = \sum_j \theta_j (1-\sigma) \left(\lambda_{ei} - \omega_{ei}^* v_j\right) = 0$$

Hence, we have that

$$\begin{aligned}
\sum_j \theta_j \lambda_{ei} &= \omega_{ei}^* \sum_j \theta_j v_j \\
\lambda_{ei} \sum_j \theta_j &= \omega_{ei}^* \sum_j \theta_j v_j \\
\lambda_{ei} &= \omega_{ei}^* \sum_j \theta_j v_j \\
\frac{\lambda_{ei}}{\omega_{ei}^*} &= \sum_j \theta_j v_j
\end{aligned}$$

Substituting this into the expression for the change in labor share, we have that when $\sigma > 1$, labor share increases whenever

$$v_j > \sum_j \theta_j v_j$$

For simplicity, let's assume $\theta_j = \frac{1}{N}$ so that labor is uniformly distributed across all sectors. This implies labor shares increase in sectors for which

$$v_j > \frac{1}{N} \sum_j v_j = \bar{v}$$

To understand why wages increase from reallocation, we start by decomposing industry output per worker. From the labor demand function, industry output per work is given by

$$h_i = \frac{y_i}{L_i} = \frac{1}{\gamma_i}\left(\frac{p_i}{w}\right)^{-\sigma}$$

Hence, h_i adjusts to the efficiency shock along two dimensions. The first dimension is a change in output y_i holding labor inputs fixed. The second is from a pure reallocation effect, holding constant output levels. Formally, industry output per worker (labor productivity) responds to the efficiency shock as follows

$$\frac{\partial h_i}{\partial \phi_{ei}} = \frac{\partial y_i}{\partial \phi_{ei}}\frac{1}{L_i} - \frac{y_i}{L_i^2}\frac{\partial L_i}{\partial \phi_{ei}}$$

The first term on the RHS is written as

$$\frac{\partial y_i}{\partial \phi_{ei}}\frac{1}{L_i} = \frac{\partial p_i^{-\sigma}}{\partial \phi_{ei}}\frac{1}{p_i^{-\sigma}}h_i + h_i\frac{\partial C}{\partial \phi_{ei}}\frac{1}{C} + h_i\frac{\partial \mathbf{Y}_i}{\partial \phi_{ei}}\frac{1}{\mathbf{Y}_i}$$

The second term on the RHS is written as

$$h_i\frac{\partial L_i}{\partial \phi_{ei}}\frac{1}{L_i} = h_i\left(\frac{\partial \mathbf{Y}_i}{\partial \phi_{ei}}\frac{1}{\mathbf{Y}_i} + \frac{\partial w^{-\sigma}}{\partial \phi_{ei}}\frac{1}{w^{-\sigma}} + \frac{\partial C}{\partial \phi_{ei}}\right)$$

Combining terms, we have

$$\frac{\partial h_i}{\partial \phi_{ei}} = h_i\left(\frac{\partial p_i^{-\sigma}}{\partial \phi_{ei}}\frac{1}{p_i^{-\sigma}} - \frac{\partial w^{-\sigma}}{\partial \phi_{ei}}\frac{1}{w^{-\sigma}}\right)$$

However, because $p_i^{-\sigma} = \mathbf{\Delta}_i^{\frac{\sigma}{\sigma-1}}w^{-\sigma}$, the expression for the change in output per worker becomes

$$\begin{aligned}\frac{\partial h_i}{\partial \phi_{ei}} &= h_i\left[\frac{\partial\left(\mathbf{\Delta}_i^{\frac{\sigma}{\sigma-1}}w^{-\sigma}\right)}{\partial \phi_{ei}}\frac{1}{\mathbf{\Delta}_i^{\frac{\sigma}{\sigma-1}}w^{-\sigma}} - \frac{\partial w^{-\sigma}}{\partial \phi_{ei}}\frac{1}{w^{-\sigma}}\right]\\ &= h_i\frac{\partial \mathbf{\Delta}_i^{\frac{\sigma}{\sigma-1}}}{\partial \phi_{ei}}\frac{1}{\mathbf{\Delta}_i^{\frac{\sigma}{\sigma-1}}}\\ &= \frac{\sigma}{\sigma-1}h_i\frac{\partial \mathbf{\Delta}_i}{\partial \phi_{ei}}\frac{1}{\mathbf{\Delta}_i}\end{aligned}$$

We can use this expression to link sector changes in output per worker to an increase in the wage rate, or changes in aggregate productivity (recall, $w = \frac{GDP}{\bar{L}}$). We illustrated earlier that wages can be expressed as

$$\frac{\partial w}{\partial \phi_{ei}} = \frac{w}{\sigma - 1} \sum_i \frac{\partial \mathbf{\Delta}_i}{\partial \phi_{ei}} \frac{1}{\mathbf{\Delta}_i} \epsilon_i$$

We can use the expression for industry output per worker $\frac{\partial h_i}{\partial \phi_{ei}}$ to show why wage rates go up. Re-arranging the expression for industry output per worker, we have

$$\frac{\partial \mathbf{\Delta}_i}{\partial \phi_{ei}} \frac{1}{\mathbf{\Delta}_i} = \frac{\sigma - 1}{\sigma} \frac{\partial h_i}{\partial \phi_{ei}} \frac{1}{h_i}$$

Substituting this into the expression for the changes in wages, we have

$$\frac{\partial w}{\partial \phi_{ei}} = \frac{w}{\sigma} \sum_i \frac{\partial h_i}{\partial \phi_{ei}} \frac{\epsilon_i}{h_i}$$

It should be noted the final expression above is a variant of Hulten's (1978) theorem. We can write this in log terms as,

$$\frac{d \log(w)}{d \log(\phi_{ei})} = \frac{1}{\sigma} \sum_i \frac{d \log(h_i)}{d \log(\phi_{ei})} \epsilon_i$$

or, as

$$d \log(w) = \frac{1}{\sigma} \sum_i \epsilon_i \, d \log(h_i)$$

From the proof of the scale effect we have

$$\lambda_{ei} = \frac{\partial w}{\partial \phi_{ei}} \frac{\phi_{ei}}{w}$$

which implies

$$\lambda_{ei} = \frac{d \log(w)}{d \log(\phi_{ei})} = \frac{1}{\sigma} \sum_i \frac{d \log(h_i)}{d \log(\phi_{ei})} \epsilon_i$$

■

Proof of Lemma 2. Partial equilibrium energy savings holds factor and commodity prices constant. Hence, the price and scale channel are shut down from any adjustments caused by the energy efficiency improvement. This implies energy savings are given by

$$\mathcal{S}_{partial} = -\frac{dy_e}{d\phi_{ei}}\phi_{ei} = -\frac{dx_{ei}}{d\phi_{ei}}\phi_{ei}$$

The impact on intermediate demand for energy in the source sector is given by

$$\frac{dx_{ei}}{d\phi_{ei}}\phi_{ei} = \underbrace{\frac{\partial\left(\frac{\omega_{ei}}{\phi_{ei}}\right)}{\partial\phi_{ei}} p_{si}^{-\sigma} p_i^{\sigma} y_i \phi_{ei}}_{\text{Technique Effect}} + \underbrace{\left(\frac{\omega_{ei}}{\phi_{ei}}\right)\frac{\partial\left(\frac{p_e}{\phi_{ei}}\right)^{-\sigma}}{\partial\phi_{ei}} p_i^{\sigma} y_i \phi_{ei}}_{\text{Energy Service Price Effect}}$$

Solving this expression yields the expression for energy savings given in Lemma 2. ■

Proof of Lemma 3. To see how the network of input-output linkages impacts the partial equilibrium effect, consider that fact that we can re-write conditional intermediate demand for energy as

$$x_{ei} = \left(\frac{\omega_{ei}}{\phi_{ei}}\right)\left(\frac{p_e}{\phi_{ei}}\right)^{-\sigma} C\mathbf{Y}_i$$

Hence, if the source sector is also an implicit supplier of energy services, i.e. the sector holds an upstream position from the energy sector, then we have the following

$$\frac{dx_{ei}}{d\phi_{ei}}\phi_{ei} = \underbrace{\frac{\partial\left(\frac{\omega_{ei}}{\phi_{ei}}\right)}{\partial\phi_{ei}} p_{si}^{-\sigma} p_i^{\sigma} y_i \phi_{ei}}_{\text{Technique Effect}} + \underbrace{\left(\frac{\omega_{ei}}{\phi_{ei}}\right)\frac{\partial\left(\frac{p_e}{\phi_{ei}}\right)^{-\sigma}}{\partial\phi_{ei}} p_i^{\sigma} y_i \phi_{ei}}_{\text{Energy Service Price Effect}} + \underbrace{\left(\frac{\omega_{ei}}{\phi_{ei}}\right)\left(\frac{p_e}{\phi_{ei}}\right)^{-\sigma} C\frac{\partial\mathbf{Y}_i}{\partial\phi_{ei}}\phi_{ei}}_{\text{Composition Effect}}$$

The full adjustment in the economy's input-output network, and how this translates into fluctuations in energy use is summarized through alterations in the input-output architecture of the economy. The vector of supplier centralities is given by

$$\mathbf{Y} = \boldsymbol{\alpha} + \boldsymbol{\Omega}^*\mathbf{Y}$$

After applying the energy efficiency shock, the vector of supplier centralities adjusts in the following manner

$$\frac{\partial \mathbf{Y}}{\partial \phi_{ei}}\phi_{ei} = \frac{\partial \mathbf{\Omega}^*}{\partial \phi_{ei}}\mathbf{Y}\phi_{ei} + \mathbf{\Omega}^* \frac{\partial \mathbf{Y}}{\partial \phi_{ei}}\phi_{ei}$$

The first term in this expression nests the partial equilibrium effect without the composition effect. In particular, carrying out the computations shows the first term yields the following

$$\frac{\partial \mathbf{\Omega}^*}{\partial \phi_{ei}}\mathbf{Y}\phi_{ei} = (\sigma - 1)\, \omega_{ei}^* \begin{bmatrix} \mathbf{Y}_i \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Hence, only the energy sector's output is affected by this first order term. In other words, we can write the change in energy sector's output from this first order effect as

$$\begin{aligned} \frac{\partial y_e}{\partial \phi_{ei}}\phi_{ei} &= p_e^{-\sigma} C \left[\frac{\partial \mathbf{\Omega}^*}{\partial \phi_{ei}}\mathbf{Y}\phi_{ei}\right]_e \\ &= (\sigma - 1)\, \omega_{ei}^* p_e^{-\sigma} C \mathbf{Y}_i \\ &= (\sigma - 1)\, x_{ei} \\ &= -\mathcal{S}_{partial} \end{aligned}$$

which shows this first order effect is partial equilibrium energy savings in the absence of input-output linkages. However, when sectors interact in the input-output network, the second term $\mathbf{\Omega}^* \frac{\partial \mathbf{Y}}{\partial \phi_{ei}}\phi_{ei}$ must be considered. By solving this expression, we find we can express the change in supplier centralities as

$$\frac{\partial \mathbf{Y}}{\partial \phi_{ei}} = \mathbf{M}\frac{\partial \mathbf{\Omega}^*}{\partial \phi_{ei}}\mathbf{Y}$$

This last expression shows how the partial equilibrium adjustment cascades throughout the network and impacts output levels for implicit providers of energy services. ∎

Proof of Proposition 5. This proof follows from the fact that energy savings from the composition channel can be expressed as function of $\mathcal{S}_{partial}$. ∎

Proof of Proposition 6. The proof of this proposition follows from the definition of the rebound effect. The rebound effect is computed as

$$\mathcal{R} = 1 - \frac{\mathcal{S}_{act}}{\mathcal{S}_{pot}}$$

In this definition, actual energy savings $\mathcal{S}_{act}$ are contrasted with potential energy savings $\mathcal{S}_{pot}$. Potential energy savings are sometimes referred to as the engineering estimate for energy savings. For example, if a device is 10% more energy efficient, then the potential energy savings is $0.1x_{ei}$. Hence, we can write the general equilibrium rebound effect as

$$\begin{aligned}\mathcal{R}_{GE} &= 1 - \frac{\mathcal{S}_{price} + \mathcal{S}_{scale} + \mathcal{S}_{comp}}{\mathcal{S}_{pot}} \\ &= 1 + (\sigma - 1)\, v_e + \frac{\sigma}{s_{ei}} \omega^*_{ei} \delta_{ie} + (1 - \sigma)\, \lambda_e\end{aligned}$$

where we take advantage of the fact that $\lambda_{ei} y_e = \lambda_e x_{ei}$ ■

Proof of Corollary 1. The composition channel can generate negative rebound effects, i.e. super conservation. Energy savings from the composition effect can be expressed as a function of partial equilibrium savings. That is,

$$\mathcal{S}_{comp} = v_e \mathcal{S}_{partial}$$

Rebound from the composition channel is expressed as

$$\begin{aligned}\mathcal{R}_{comp} &= 1 - \frac{\mathcal{S}_{comp}}{\mathcal{S}_{pot}} \\ &= 1 - v_e \frac{\mathcal{S}_{partial}}{\mathcal{S}_{pot}}\end{aligned}$$

We can take advantage of the fact that partial equilibrium rebound is expressed as

$$\mathcal{R}_{partial} = 1 - \frac{\mathcal{S}_{partial}}{\mathcal{S}_{pot}}$$

which implies

$$\frac{\mathcal{S}_{partial}}{\mathcal{S}_{pot}} = 1 - \mathcal{R}_{partial}$$

Substituting this into rebound from the composition effect, we have

$$\mathcal{R}_{comp} = 1 - v_e \left(1 - \mathcal{R}_{partial}\right)$$

Hence, we can actually now derive conditions for when super conservation occurs from the composition channel. First, we solve for the actual partial equilibrium rebound effect.

$$\begin{aligned}\mathcal{R}_{partial} &= 1 - \frac{\mathcal{S}_{partial}}{\mathcal{S}_{pot}} \\ &= 1 - \frac{(1-\sigma)\, x_{ei}}{x_{ei}} \\ &= \sigma\end{aligned}$$

Second, we consider the case when $\sigma < 1$ since Proposition 7 stipulates energy savings from the composition effect will be larger than the partial equilibrium savings in this case, which could potentially generate super conversation, or negative rebound. Super conservation occurs when

$$\begin{aligned}\mathcal{R}_{comp} &< 0 \\ 1 - v_e \left(1 - \mathcal{R}_{partial}\right) &< 0 \\ \frac{1}{1 - \mathcal{R}_{partial}} &< v_e\end{aligned}$$

By substituting the value for $\mathcal{R}_{partial}$, we find that super conservation occurs when

$$v_e > \frac{1}{1-\sigma} \tag{27}$$

■

Appendix B Calibration Details

The production system is characterized by exogenously given input-output relationships between sectors $\boldsymbol{\Omega}$, the share of labor in each sector γ, and the share of each sector's output in final consumption $\boldsymbol{\alpha}$. To calibrate these parameters of the model using the IMPLAN data, we normalize industry prices $\mathbf{P}$, the economy's wage rate w, the economy's labor force $\bar{L}$, and the consumer price index P_c to be equal to 1. Additionally, we take the across industry elasticity of substitution σ to be a known constant. Given this normalization, the model can be calibrated using the available input-output data for each state.

B.1 The Input-Output Network

Consider the equation for intermediate demand given in equation (6a). Assume $\phi_{ei} = 1$ and re-arrange this equation to solve for the input-output coefficient ω_{ei}. Under the steady-state condition $\mathbf{P} = \mathbf{1}$, we have the following identity that is used to calibrate the input-output matrix

$$\begin{aligned}\omega_{ei} &= \frac{p_e^{\sigma} x_{ei}}{p_i^{\sigma} y_i} \\ &= a_{ei}^{\sigma} \left(\frac{p_e x_{ei}}{p_i y_i} \right)^{1-\sigma} \\ &= d_{ei}\end{aligned}$$

Hence, we can use the direct requirements matrix $\mathbf{D}$ calculated from the IMPLAN data to calibrate the input-output coefficient matrix $\boldsymbol{\Omega}$ in the model. Importantly, in the simulation, energy efficiency shocks will be applied to the direct requirements matrix $\mathbf{D}$ to simulate changes in energy production for each state.

B.2 Labor Shares

We also use the IMPLAN data to calibrate the labor share parameters $\boldsymbol{\gamma}$ in the model. The approach is similar to the method for calibrating the input-output matrix. Equilibrium in the model implies the conditional factor demand for labor in sector i is given by equation (**??**). Given $w = 1$ and $\mathbf{P} = \mathbf{1}$ in the steady-state, re-arranging the expression for conditional factor demand to solve for the labor share parameter γ_i implies the following

$$\begin{aligned}\gamma_i &= \frac{w^{\sigma} L_i}{p_i^{\sigma} y_i} \\ &= g_i^{\sigma} \left(\frac{w L_i}{p_i y_i}\right)^{1-\sigma} \\ &= g_i\end{aligned}$$

Thus, by assuming wages and sector prices are equivalent to their steady-state values, we can use the labor expenditure shares for the sectors, denoted as an $N \times 1$ vector $\mathbf{g}$, to calibrate the labor intensity parameters $\boldsymbol{\gamma}$ in the model.

B.3 Consumption Shares

The final set of model parameters, consumption shares $\boldsymbol{\alpha}$, are calibrated using a similar approach as above. Consider the expression for final consumption in equation (2). Rearranging this equation to solve for the consumption share of sector i and incorporating the steady-state condition for prices implies

$$\begin{aligned}\alpha_i &= \frac{p_i^{\sigma} c_i}{C} \\ &= a_i^{\sigma} \left(\frac{p_i c_i}{C}\right)^{1-\sigma} \\ &= a_i\end{aligned}$$

Given the steady-state conditions, I can use the household's budget shares, denoted as the $N \times 1$ vector $\mathbf{a}$, to calibrate the household preferences parameters in the model.

B.4 Simulation

Once the model is calibrated to data, we can compute the multiplier matrix of the economy and simulate the effects from an energy efficiency shock. For the simulation, we consider efficiency improvements that affect production in three energy-related sectors:

(i) Coal Mining (NAICS 212111-212113), (ii) Petroleum Refineries (NAICS 324110), and (iii) Natural gas distribution (NAICS 221210).

Energy production is computed using the calibrated model parameters. Given the underlying data is in steady-state, baseline energy production levels are computed as

$$\tilde{y}_e^0 = [\mathbf{I} - \mathbf{D}]_e^{-1} \mathbf{a} = \tilde{\mathbf{Y}}_e \tag{28}$$

where $\tilde{y}_e^0$ is the simulated, baseline production for each energy sector e, and $\tilde{\mathbf{Y}}_e$ is the energy sector's simulated supplier centrality measure. In the simulation, I apply a 10% energy efficiency improvement to each sector by shocking the direct requirements matrix. Importantly, I only consider one shock at a time. The productivity-adjusted direct requirements matrix $\boldsymbol{\phi}^{\sigma-1} \odot \mathbf{D} = \mathbf{D}^*$ accounts for the 10% energy efficiency improvement through the multiplication of $\boldsymbol{\phi}^{\sigma-1}$ and $\mathbf{D}$. Specifically, the e, i-th entry in $\mathbf{D}^*$ becomes $(1.1)^{\sigma-1}\omega_{ei}$ for a given value of σ after the efficiency shock is applied

Energy production after the shock is computed by replacing the direct requirements matrix $\mathbf{D}$ in equation (28) with the productivity-adjusted version. However, the efficiency shock will have the effect of shocking prices and household income from their steady-state values. This implies energy production after the shock can be computed numerically using the following expression

$$\tilde{y}_e^1 = \frac{\tilde{\boldsymbol{\Delta}}_e^{*\frac{\sigma}{\sigma-1}} \tilde{\mathbf{Y}}_e^*}{\tilde{\mathbf{Y}}^{*\prime} \mathbf{g}} \tag{29}$$

where $\tilde{y}_e^1$ is the simulated energy production after the shock, $\tilde{\boldsymbol{\Delta}}_e^*$ is the energy sector's simulated consumer centrality after the shock, and $\tilde{\mathbf{Y}}^*$ is the simulated vector of supplier centrality in the economy. After new energy production is computed, energy savings in the model can be calculated by taking the difference in energy production before and after the shock. Aggregate energy savings $\tilde{S} = y_e^0 - y_e^1$ is then computed by applying shocks to the sectors mentioned above for each state.

Appendix C Additional Simulation Results

C.1 Channels

We start by providing information on the fraction of variation in rebound explained by each general equilibrium channel. Table 3 reports the fraction of variation in simulated general equilibrium rebound effect explained by each channel. The table summarizes the result for each energy sector and elasticity of substitution. The reported results illustrate that within the same model framework, i.e. same elasticity of substitution, the price channel tends to explain a majority of variation in simulated rebound effects.

Table 3: Fraction of Variation in Simulated Rebound Explained by General Equilibrium Channels

	All ($N = 303,408$)			Coal ($N = 48,012$)			Oil ($N = 126,478$)			Gas ($N = 128,928$)		
	Price	Scale	Composition	Price	Scale	Composition	Price	Scale	Composition	Price	Scale	Composition
$\sigma = 0.25$	0.395	0.168	0.437	0.303	0.528	0.170	0.490	0.216	0.293	0.990	0.005	0.005
$\sigma = 0.5$	0.887	0.011	0.103	0.837	0.117	0.046	0.928	0.010	0.061	0.997	0.002	0.001
$\sigma = 0.75$	0.984	0.004	0.012	0.987	0.007	0.006	0.987	0.006	0.007	0.999	0.001	0.000
$\sigma = 1.25$	0.957	0.038	0.004	0.979	0.019	0.002	0.948	0.049	0.002	0.999	0.001	0.000
$\sigma = 1.50$	0.937	0.051	0.012	0.958	0.037	0.005	0.928	0.065	0.007	0.999	0.001	0.000
$\sigma = 1.75$	0.920	0.061	0.019	0.940	0.051	0.009	0.912	0.077	0.011	0.999	0.001	0.000
All	0.025	0.002	0.973	0.045	0.001	0.954	0.029	0.003	0.968	0.014	0.006	0.981

To understand how the network component drives variation in the rebound effect, we compute the explained variation from a regression of the rebound effect from the price channel on the downstream percolation centrality of the source sector. The results of this exercise are reported in Table 4. The results suggest the network component of the price channel explains nearly 50% of total variation in rebound from the price effect. Within each simulation, we find the total variation explained by the network component could reach as high as 91.8 percent.

Table 4: Fraction of Variation in Price Channel Explained by the Network Component

	All ($N = 303,408$)	Coal ($N = 48,012$)	Oil ($N = 126,478$)	Gas ($N = 128,928$)
	Price Effect	Price Effect	Price Effect	Price Effect
$\sigma = 0.25$	0.623	0.648	0.483	0.918
$\sigma = 0.5$	0.623	0.648	0.483	0.918
$\sigma = 0.75$	0.623	0.649	0.483	0.918
$\sigma = 1.25$	0.624	0.650	0.484	0.918
$\sigma = 1.50$	0.624	0.650	0.484	0.918
$\sigma = 1.75$	0.624	0.650	0.484	0.918
All	0.475	0.493	0.369	0.699

C.1.1 The Price Effect

Figure 6 visualizes the results for the simulations where $\sigma = 0.5$. We find the potential for backfire from the price channel is significant even at this value for the elasticity of substitution. In these cases, the input cost effect dominates the value added effect, leading to a net reduction in the price for energy. As the energy price falls, aggregate energy use will increase above the baseline level. From the figure, we are able to conjecture the potential for backfire increases as one moves into the upper tail of the downstream percolation centrality distribution. Since we are visualizing the natural logarithm of downstream percolation centrality, the approximately normal distribution in the figure implies the underlying centrality distribution is log-normal with a fat upper tail. Hence, the distribution of downstream percolation centrality potentially explains the fat tailed distribution of simulated rebound effects from the price channel. The implication is that energy efficiency investments in sectors with a higher value of δ_{ie} are more likely to lead to larger reduc-

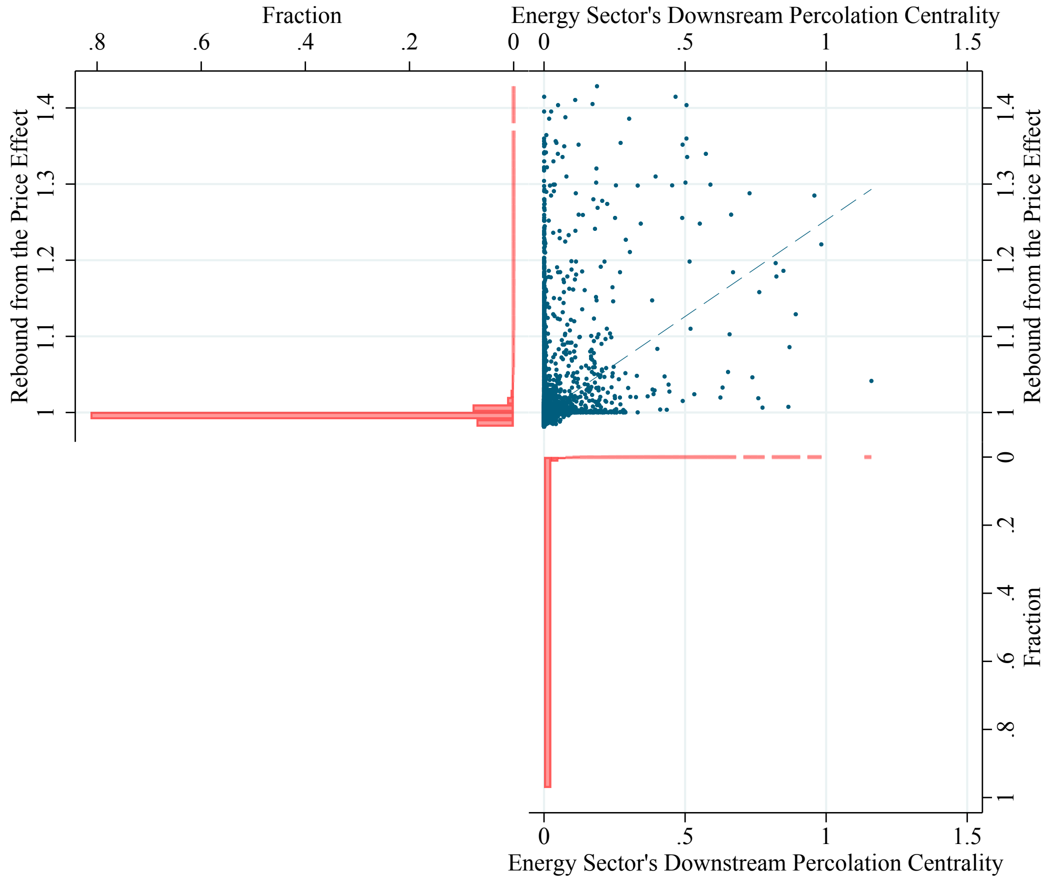

Figure 6: Simulated Rebound from the Price Channel. The figure plots the simulated rebound effects from the price channel and the histogram of these effects. We superimpose the distribution of percolation centralities to relate this distribution to the distribution of simulated rebound.

tions in the real price for energy than investments in sectors where δ_{ie} lies in the bottom portion of the distribution.

C.1.2 The Scale Effect

The simulations predict the scale effect leads to backfire as suggested by theory. The typical simulated rebound effects are 1.003 for Coal, 1.004 for Gas, and 1.013 for Oil. Across all simulations, simulated rebound from the scale effect can differ by as much as 4%. The source of this variation is attributable to the distribution of Domar weights for energy sector's across states. Figure 7 plots the results from the simulation along with the distribution of Domar weights for the energy sector. The figure shows simulated rebound

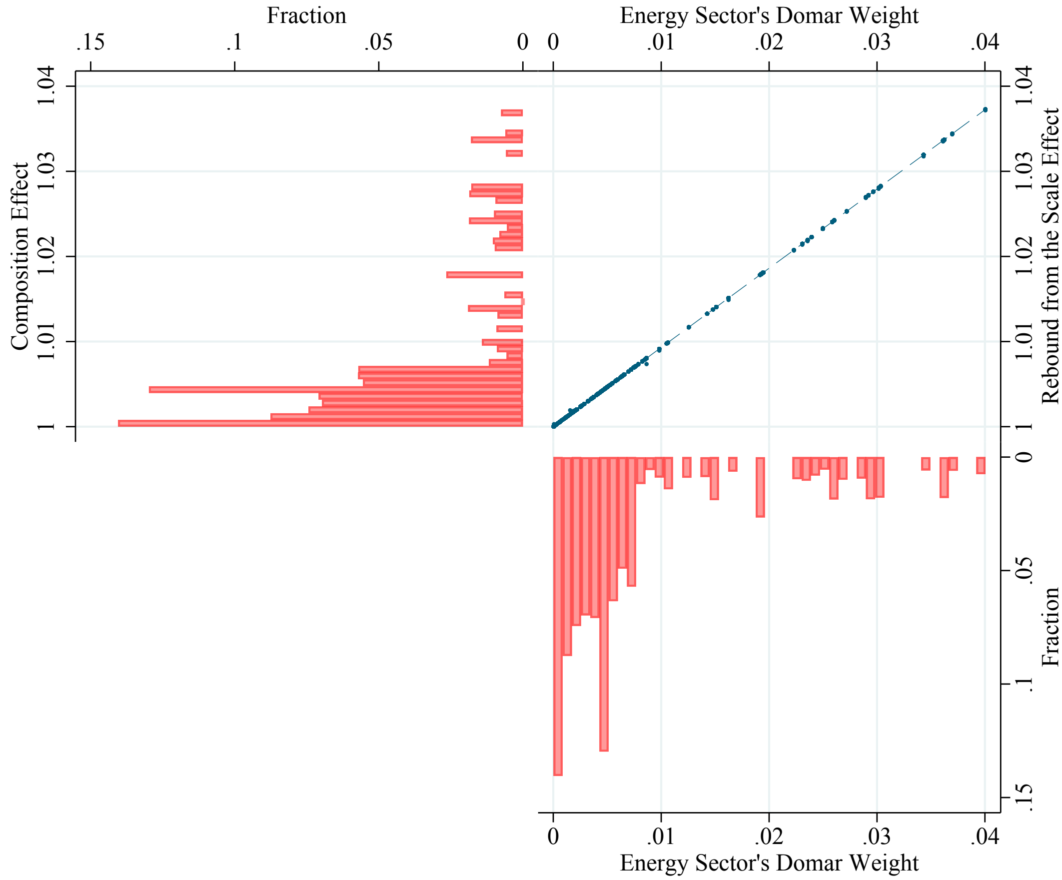

Figure 7: Simulated Rebound from the Scale Effect. The figure plots the simulations results for rebound driven by the scale effect. The distribution of rebound effects and Domar weights are superimposed on the figure for comparison.

from the scale effect accords with the theoretical predictions. We have binned the observations using 50 bins to coincide with the number of U.S. states. The figure shows for the vast majority of US states the sales share of the energy sector is less than 1%. However, in some cases, the sales shares could rise to higher than 3%, indicating these states may experience large rebound from energy efficiency investments. Representative examples include the oil sectors in both Louisiana ($\lambda_e = 3.6\%$) and Wyoming ($\lambda_e = 3.6\%$).

C.1.3 The Composition Effect

We plot the results from the simulation where $\sigma = 0.5$ in Figure 8. The figure shows the relationships between the distribution of v_e (bottom), rebound from the composition

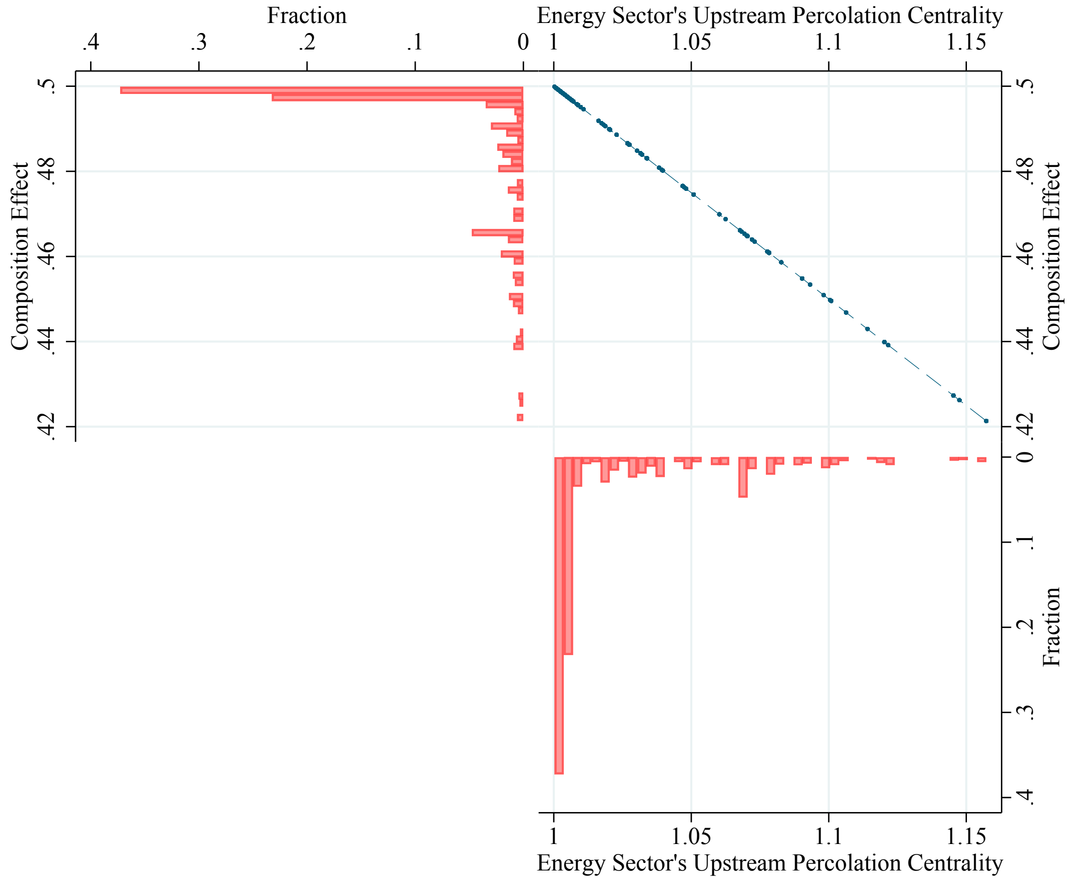

Figure 8: Simulated Rebound from the Composition Effect. In this figure, we plot the simulation results for rebound caused by the composition effect for $\sigma = 0.5$. We also provide the histograms for the rebound effect and the energy sector's upstream percolation centrality.

effect $\mathcal{R}_{comp}$ (top right), and the distribution of rebound from the composition effect (top left). Because $\sigma < 1$ in this simulation, the rebound effect is a decreasing function of v_e, which accords with the predictions of the theoretical model. In the data, the energy sector's upstream percolation centrality ranges from a little more than 1 to 1.15, which translates into rebound that ranges between 0.42 and 0.50. The figure illustrates how the distribution of v_e dictates the magnitude of the rebound effect. In more than 60% of the simulations, rebound from the composition channel approximately coincides with the partial equilibrium prediction. However, in rarer cases, the energy sector's position in the network creates a multiplier effect on partial equilibrium energy savings, driving rebound to around 16% lower than the partial equilibrium prediction would suggest.

Representative examples of these rarer cases include energy efficiency improvements that impact the coal sectors in West Virginia ($v_e = 1.145$) and Wyoming ($v_e = 1.148$).

Appendix D A Model with Capital Inputs

We extend the setup in the paper to include a capital input. The capital input enters into a value added nest $V = V(L, K)$ where substitution between labor and capital takes place. Incorporating this value added nest into the original production function, we have the nested production function is expressed as

$$y_i = \left[\gamma_i^{\frac{1}{\sigma}} V_i^{\frac{\sigma-1}{\sigma}} + \omega_{ei}^{\frac{1}{\sigma}} \left(\phi_{ei} x_{ei} \right)^{\frac{\sigma-1}{\sigma}} + \sum_j \omega_{ji}^{\frac{1}{\sigma}} x_{ji}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

To maintain tractability, we assume the value added nest is Cobb-Douglas of the form

$$V_i = V(K_i, L_i) = L_i^{\lambda} K_i^{1-\lambda}$$

where $\lambda \in (0, 1)$. Assuming producers cost minimize, the first order conditions imply the conditional demand for labor and capital are given by

$$L_i = G^{\lambda-1} \left(\frac{w}{r} \right)^{\lambda-1} V_i$$
$$K_i = G^{\lambda} \left(\frac{w}{r} \right)^{\lambda} V_i$$

where $G(\lambda) = \frac{1-\lambda}{\lambda}$ is a function of parameters. Using these expressions, we can solve for value added expenditures as

$$p_V V_i = w L_i + r K_i = H w^{\lambda} r^{1-\lambda} V_i$$

where $H = \frac{1}{\lambda} G^{\lambda}$, and the value added price index for producers is given by

$$p_V = H w^{\lambda} r^{1-\lambda}$$

The first-order conditions for the top level production function imply the following:

1. **Value Added:** $V_i = \gamma_i p_V^{-\sigma} p_i^{\sigma} y_i$
2. **Energy Input:** $x_{ei} = \omega_{ei}^* p_e^{-\sigma} p_i^{\sigma} y_i$
3. **Non-energy Input:** $x_{ji} = \omega_{ji} p_j^{-\sigma} p_i^{\sigma} y_i$

We combine these first order conditions to solve for the output price for each industry. Output prices are given by

$$p_i = \left[\gamma_i p_V^{1-\sigma} + \omega_{ei}^* p_e^{1-\sigma} + \sum_j \omega_{ji} p_j^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

With these expressions, we can solve for expenditures as function of the value added price index, marginal costs, and output

$$\begin{aligned} wL_i &= G^{\lambda-1} p_V^{1-\sigma} \gamma_i p_i^{\sigma} y_i \\ rK_i &= G^{\lambda} p_V^{1-\sigma} \gamma_i p_i^{\sigma} y_i \end{aligned}$$

Using the market clearing conditions, we find the following

$$\frac{w}{r} = \frac{\lambda}{1-\lambda} \tilde{k}$$

where $\tilde{k} = \frac{\bar{K}}{\bar{L}}$ is the per capita capital stock. Using this result, conditional demand for labor and capital can be written as

$$\begin{aligned} L_i &= \tilde{k}^{\lambda-1} V_i \\ K_i &= \tilde{k}^{\lambda} V_i \end{aligned}$$

By substituting the expression for w/r into the producer's value added price index, we obtain the following

$$\begin{aligned} p_V &= H\left(\frac{w}{r}\right)^{\lambda} r \\ &= \frac{1}{\lambda} G^{\lambda}\left(\frac{\tilde{k}}{G}\right)^{\lambda} r \\ &= \frac{1}{\lambda}\tilde{k}^{\lambda} r \end{aligned}$$

Substituting this expression for p_v into the expression for V_i and deriving the conditional labor and capital demand functions, we find

$$\begin{aligned} L_i &= \lambda^{\sigma}\tilde{k}^{\lambda-1-\lambda\sigma} r^{-\sigma}\gamma_i p_i^{\sigma} y_i \\ K_i &= \lambda^{\sigma}\tilde{k}^{\lambda-\lambda\sigma} r^{-\sigma}\gamma_i p_i^{\sigma} y_i \end{aligned}$$

The market clearing condition for capital requires that

$$\bar{K} = \lambda^{\sigma}\tilde{k}^{\lambda-\lambda\sigma} r^{-\sigma}\sum_i \gamma_i p_i^{\sigma} y_i$$

Taking advantage of the fact that $\mathbf{Y}_i = \frac{p_i^{\sigma} y_i}{C}$, we can express the capital market clearing condition as

$$\bar{K} = \lambda^{\sigma}\tilde{k}^{\lambda-\lambda\sigma} r^{-\sigma} C\mathbf{Y}'\gamma$$

Total income (GDP) in the economy is given by

$$GDP = C = w\bar{L} + r\bar{K}$$

which implies GDP per capita is expressed as

$$\frac{C}{\bar{L}} = w + r\tilde{k}$$

However, we can utilize the fact that $w = \frac{\lambda}{1-\lambda} r\tilde{k}$ to solve for an expression for GDP per capita that is only in terms of r. Formally, we have

$$\begin{aligned}\frac{C}{\bar{L}} &= w + r\tilde{k} \\ &= \frac{\lambda}{1-\lambda} r\tilde{k} + r\tilde{k} \\ &= \frac{1}{1-\lambda} r\tilde{k}\end{aligned}$$

Above we found that we can express the capital stock as a function of network centrality concepts, i.e. $\bar{K} = \lambda^{\sigma} \tilde{k}^{\lambda - \lambda\sigma} r^{-\sigma} C \mathbf{Y}' \boldsymbol{\gamma}$. To arrive at a closed-form expression for r, start by dividing both sides of the capital market clearing condition by $\bar{L}$ to yield

$$\tilde{k} = \lambda^{\sigma} \tilde{k}^{\lambda - \lambda\sigma} r^{-\sigma} \frac{C}{\bar{L}} \mathbf{Y}' \boldsymbol{\gamma}$$

Replace $\frac{C}{\bar{L}}$ with $\frac{\lambda}{1-\lambda} r\tilde{k}$ to arrive at

$$\tilde{k} = \frac{\lambda^{\sigma}}{1-\lambda} \tilde{k}^{\lambda - \lambda\sigma} r^{1-\sigma} \tilde{k} \mathbf{Y}' \boldsymbol{\gamma}$$

Combining like terms and re-arranging the above expression to solve for r gives the following

$$r = \lambda^{\frac{\sigma}{\sigma - 1}} (1-\lambda)^{\frac{1}{1-\sigma}} \tilde{k}^{-\lambda} \left(\mathbf{Y}' \boldsymbol{\gamma} \right)^{\frac{1}{\sigma - 1}} \tag{30}$$

The expression dictates that the rental rate of capital is decreasing in $\tilde{k}$ and inherently depends on the network characteristics. Intuitively, the model dictates that capital deepening drives down rental rates as capital becomes relative more abundant per worker. Lastly, combining this result with the expression for w we have

$$\begin{aligned}w &= \frac{\lambda}{1-\lambda} r\tilde{k} \\ &= \frac{\lambda}{1-\lambda} \lambda^{\frac{\sigma}{\sigma-1}} (1-\lambda)^{\frac{1}{1-\sigma}} \tilde{k}^{1-\lambda} (\mathbf{Y} \boldsymbol{\gamma})^{\frac{1}{\sigma-1}} \\ &= F(\lambda, \sigma) \tilde{k}^{1-\lambda} \left(\mathbf{Y}' \boldsymbol{\gamma} \right)^{\frac{1}{\sigma-1}}\end{aligned}$$

Furthermore, the price index for value added services is given by

$$p_V = \frac{1}{\lambda}\tilde{k}^{\lambda} r = \lambda\left(1-\lambda\right)^{\frac{1}{1-\sigma}}\left(\mathbf{Y}'\gamma\right)^{\frac{1}{\sigma-1}}$$

D.1 Equilibrium Prices and the Price Effect

The above results show that adding capital to the model, along with a value added nest, do not alter the mechanics of the rebound effect presented in the paper. To see this formally, start by solving for equilibrium prices in the model. Output prices are given by

$$p_i = \left[\gamma_i p_V^{1-\sigma} + \omega_{ei}^{*} p_e^{1-\sigma} + \sum_j \omega_{ji} p_j^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Substituting the expression for the value added index implies the vector of output prices are given by

$$\mathbf{P} = \mathbf{\Delta}^{\frac{1}{1-\sigma}}\left[\lambda\left(1-\lambda\right)^{\frac{1}{1-\sigma}}\left(\mathbf{Y}'\gamma\right)^{\frac{1}{\sigma-1}}\right]$$

where the term in brackets is a scalar. The equilibrium energy price is thus expressed as

$$p_e = \mathbf{\Delta}_e^{\frac{1}{1-\sigma}}\lambda\left(1-\lambda\right)^{\frac{1}{1-\sigma}}\left(\mathbf{Y}'\gamma\right)^{\frac{1}{\sigma-1}}$$

In the model without capital inputs, the equilibrium energy price is given by

$$p_e = \mathbf{\Delta}_e^{\frac{1}{1-\sigma}} w = \mathbf{\Delta}_e^{\frac{1}{1-\sigma}}\left(\mathbf{Y}'\gamma\right)^{\frac{1}{\sigma-1}}$$

D.2 Equilibrium Income and the Scale Effect

Now that we have illustrated the mechanics of the price effect will be the same, we turn our attention to the mechanics of the income effect. In the capital input model, equilibrium income can be expressed as

$$C = w\bar{L} + r\bar{K} = f\left(\lambda,\sigma,\tilde{k}\right)\left(\mathbf{Y}'\gamma\right)^{\frac{1}{1-\sigma}}$$

Since $f\left(\lambda, \sigma, \tilde{k}\right)$ consists of constants, this quantity will not adjust following the efficiency shock. Instead, we can see the mechanics are identical to what is found in the paper. Again, we note the magnitude of the rebound effect is different with capital inputs.